Retailer- vs. Vendor-Managed Inventory and Brand Competition

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Vendor-managed inventory (VMI) is emerging as a significant development in the recent trend towards collaboration and information sharing in supply chain management. Transfer of inventory monitoring and other overhead costs to manufacturers and continuous replenishment of retailer inventory are commonly cited as potential benefits that VMI offers to retailers. We provide a new explanation in this paper for why retailers might be interested in VMI. We show that VMI intensifies the competition between manufacturers of competing brands and that the increased competition benefits a retailer that stocks these brands. Competition arises because of brand substitution; that is, some consumers may switch to another brand if their "preferred" brand is out of stock. The manufacturer whose brand is out of stock thus risks losing sales from those consumers who buy the competing brand. Consequently, each manufacturer has an incentive to keep a higher stock of its own brand, not only to satisfy the demand from its customers, but also the spillover demand that arises if a competing brand goes out of stock. When the retailer makes the stocking-level decisions, the competition is mitigated by the pooling of demands at the retailer. VMI restores the competition between the manufacturers and benefits the retailer.

Key words: retailing; supply chain management; product substitution; inventory management

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1. Introduction

Consider a retailer who sells multiple brands of a product produced by different manufacturers. Who should make the decision about how many units of each brand the retailer stocks? In traditional supply chains, the retailer makes the stocking decision. This may be justified based on the argument that often the retailer has better information about consumer demand and can manage the product category better by optimizing the total stock for the product. In such retailer-managed inventory (RMI) systems, the retailer places orders with the manufacturer who fulfills these orders. More recently, vendor-managed inventory (VMI) has become popular in some supply chains. For example, manufacturers such as Campbell Soup and Proctor and Gamble and retailers such as Wal-Mart have been engaged in VMI relationships for some time. In a VMI system, the vendor or manufacturer is responsible for the management of stock at the retailer. The retailer provides the manufacturer with access to its real-time inventory level. The replenishment decisions, i.e., how much and how often to replenish, are made by the manufacturer. The retailer may set certain service-level and/or shelf space requirements that are taken into consideration by the manufacturer. That is, in a VMI system, the retailer’s role shifts from managing inventory to simply renting retailing space.

VMI is part of the trend in supply chains that encourages collaboration and information sharing between trading partners. Literature on collaboration in supply chains shows that information sharing reduces the "bullwhip" effect at the manufacturer (Lee et al. 1997, Drezner et al. 2000), and the primary beneficiary is the manufacturer. Some of these researchers have pointed out that VMI may be one way by which the retailer can get back some of this benefit because VMI can reduce a retailer's inventory-monitoring and ordering costs. In fact, many retailers are pushing manufacturers to adopt VMI in return for sharing demand information (Lee et al. 2000a). While it is true that VMI can eliminate or at least lower retailers' inventory-monitoring and ordering costs, these costs have already been reduced significantly by Electronic Data Interchange (EDI) systems that have been in place for several years. This extensive literature includes Bourland et al. (1996), Cachon (1998), Guille et al. (2000), Cervinani et al. (1999), Lee et al. (2000), Raghunathan (2001), and Raghunathan and Yeh (2001).

Other benefit-sharing arrangements might include price discounts, lead-time reduction, and better credit terms.
years and, more recently, by the Internet. Hence, the reduction of inventory-monitoring and ordering costs may not completely explain retailers’ demand that manufacturers adopt VMI.

In this paper, we show that VMI intensifies the competition between manufacturers of competing brands, and also show that the increased competition benefits a retailer that stocks these brands. This result offers another explanation for why retailers may demand that manufacturers adopt VMI. The competition between brands arises because of brand substitution; that is, consumers may switch to another brand available at the retailer if their “preferred” brand is out of stock. The manufacturer whose brand is out of stock thus risks losing sales from customers who buy competing brands. When brands from different manufacturers are substitutable, each manufacturer has an incentive to keep a higher stock of its brand, not only to satisfy the demand from its own customers, but also the spillover demand that arises if a competing brand goes out of stock. In essence, the manufacturers engage in shelf space or quantity competition. VMI, as opposed to RMI, enables the retailer to better exploit this competition between suppliers of substitutable products. When the retailer makes the stocking-level decisions, the competition is mitigated by the pooling of demands at the retailer. VMI restores the competition between manufacturers and benefits the retailer.

We perform our analysis of RMI and VMI within the context of a single retailer stocking two brands of an item, each produced by a different manufacturer. We study the impact of substitutability between these brands on the stocking levels and profits when (i) the retailer and (ii) manufacturers make the stocking decisions. The degree of substitutability between the brands is captured by a single parameter that represents the fraction of customers who will switch brands if their preferred brand is out of stock. When this parameter is zero, the brands are nonsubstitutable. A value of one for this parameter implies the brands are perfect substitutes in the sense that every consumer will buy the alternate brand when his/her preferred brand is not available.

Our insight about how the retailer, by delegating stocking decisions, can exploit the upstream competition is new to the literature. In RMI, the retailer has two variables to coordinate, i.e., price and stocking level, in order to maximize its profit that consists of three components: profit from sales, inventory holding cost, and shortage cost. However, the manufacturers do not make the stocking-level decision under RMI, and consequently, the retailer is unable to exploit quantity competition between manufacturers caused by brand substitution. In VMI, the retailer delegates the stocking-level decisions. While the retailer gives control over the stocking decisions, it also eliminates the inventory holding cost. Delegation of the stocking-level decision has another benefit for the retailer; it restores the quantity competition. The retailer then exploits this quantity competition to reduce its shortage cost and enhance his profit.

The rest of the paper is organized as follows. Section 2 reviews related research in this area. We discuss the model in §3 and present our analysis of the model and principal results in §4 for a general demand distribution. Section 5 provides additional insights for the case of uniformly distributed demands. We conclude the paper with a summary in §6.

2. Literature Review

There has been a significant body of work in the operations management literature that addresses the inventory management issues in the context of product substitutability. Some of these papers assume “one-way substitutability,” i.e., a higher-quality product can substitute a lower-quality product, but not vice versa. These papers include Bassok et al. (1999), Hsu and Bassok (1999), Bitran and Dasu (1992), Rao et al. (1998), and Pentico (1974). These papers characterize the optimal stocking or production policy and, in some cases, provide heuristics and algorithms for determining the optimal level. Another set of papers deals with developing demand models when consumers, with a dynamic arrival pattern, buy substitute products when their preferred product is out of stock (Smith and Aggrawal 2000, Mahajan and van Ryzin 2001). Other papers deal with two-way substitutability. Papers in this stream show that tractable analysis doesn’t exist for general models with two-way substitutability. McCall and Silver (1978) develop simulation analysis and heuristics for determining order quantity. Parlar and Goyal (1984) and Parlar (1988) analyze the substitutable product inventory problem with random demands. They derive the Nash equilibrium solution for the stocking levels when customers search for the item in the other retailer when the item at the preferred retailer is out of stock. Pasternack and Dreznner (1991) use numerical analysis to derive order quantities. Rajaram and Tang (2000) present an approximation that allows a tractable and accurate heuristic to compute order quantities and profits when the degree of substitution varies from zero to one. They also study how the level of demand variation and correlation and the degree of substitution affect order quantities and profits.

The substitutability issue that arises when two retailers compete with the same product has also been studied. Rudi and Netessine (1999) show that the order quantity under substitution and retailer competition is always greater than the newsboy order quantity for the base case without substitution and without
competition. Anupindi and Bassok (1999) consider the competitive dynamics between competing retailers and an upstream manufacturer. They study the issue of decentralization of stocks in a system with a single manufacturer whose product is carried by two retailers. They showed that centralization of stocks by retailers does not always benefit the manufacturer. Lippman and McCardle (1997) analyze a competitive newsboy (oligopoly) model. The competition occurs because the aggregate demand is allocated to firms using allocation rules that specify how the excess stock in one location satisfies the demand in another location. Narayanan and Raman (2000) consider the problem of stocking-decision rights and identify conditions under which stocking-decision rights should be transferred from the retailer to the manufacturer. Finally, Anupindi et al. (1999) develop a general framework for the analysis of decentralized distribution systems of \(N\) retailers who hold stocks locally and at one or more central locations.

Our paper is also related to the marketing and economics literature on the allocation of responsibility for unsold inventory, including returns policies and price protection policies (Pasternack 1985, Kandel 1996, Butz 1997, Padmanabhan and Png 1997, Lee et al. 2000). Padmanabhan and Png (1997, p. 82) "show that, by offering a returns policy, a manufacturer shifts the basis of retailer competition from quantities (Cournot) to price (Bertrand) and so intensifies the degree of competition." Our result is somewhat similar to theirs. In our problem, by delegating the stocking decision, a retailer exploits the Cournot competition between manufacturers. From the retailer's point of view, more intense Cournot competition means greater sales and/or lower shortages. However, none of this literature considers a distribution system with product substitution. Also, as opposed to downstream retailer competition considered by this literature, we consider upstream manufacturer competition.

Our research differs in many ways from earlier research noted above, and is motivated by our quest to understand the impact of VMI when products are substitutable. A significant observation that underpins our model framework is that in the retail and grocery industry, where VMI is popular, competition between brands within a store is perhaps more dominant than competition between retailers for the same brand. Thus, we consider a distribution system in which a single retailer stores substitutable brands of an item from two different manufacturers. In distribution systems considered in prior literature, centralization of stock by retailers using a common warehouse eliminates the impact of consumer search or substitution because consumer demands at multiple locations are met from this central warehouse.3 In our context, we consider an already centralized system in the sense that the consumer demand is met from a single location, i.e., retailer. However, the extent of consumer search, which is dependent on the exogenous degree of substitutability between brands, affects the supply chain profits under all inventory management scenarios.

3. Model Development

We use a two-level manufacturer-retailer setting to study the impact of RMI and VMI on the supply chain. There is a single downstream retailer that sells two brands of a product. There are two upstream manufacturers, each producing a different brand. The brands are substitutable in the sense that a customer looking for Brand 1 may buy Brand 2 if Brand 1 is out of stock, and vice versa. To clearly focus on the main issue of brand substitutability, we assume that manufacturers' cost structures, as well as brands' demand structures, are symmetric.

The demand for brand \(i\) during period \(t\), \(d_{it}\), is a function of its price \(p_{it}\) and is given by

\[
d_{it} = X_{it} - \beta p_{it}, \quad i \in \{1, 2\}, \beta > 0,
\]

where \(p_{it}\) is the retail price of brand \(i\) during period \(t\) and \(X_{it}\) is the market potential for brand \(i\) during period \(t\). \(X_{it}\) is a random variable that has a probability density function \(f(x_{it})\) with mean \(a\). We assume that \(X_{it}\)s are i.i.d. across brands for a given time period and across time periods for a given brand. We also assume that each customer demands one unit of the product.

The brand a customer is looking for ("preferred brand") may not be in stock. When his preferred brand is out of stock, a customer has two options. He may either buy the alternative brand, if it is in stock, or he may backorder his preferred brand. We model a customer’s willingness to buy the alternative brand by the parameter \(\alpha\). Various factors, such as brand loyalty and cost of searching for the preferred brand elsewhere, could determine \(\alpha\). The retailer, as well as manufacturer \(i\), incurs a shortage cost for brand \(i\) when brand \(i\) is out of stock. This shortage cost is \(s\) per unit. The shortage cost is in addition to the potential loss in profit from the lost sale of brand \(i\).

\[3\text{See Anupindi and Bassok (1999) for examples from the auto industry.}\]

\[4\text{In our model, competition between brands arises solely from substitution effect. Price competition at the retailer level does not change our results qualitatively. We discuss this in §7.}\]

\[5\text{Although one could make a convincing case of different holding costs and shortage costs for the retailer and manufacturers.}\]
We assume that the production cost for each brand is \( c \) per unit. All parameters are common knowledge to all parties. We also require that \( a > bc \), a necessary condition for the manufacturers and the retailer to realize nonnegative profits.

Price and stocking-level decisions are made before the demands are realized in both RMI and VMI. In RMI, the retailer makes the stocking-level decisions and incurs all inventory holding costs. In VMI, the manufacturers make the stocking-level decisions and incur all inventory holding costs. We first consider a scenario in which the wholesale and retail prices are exogenously fixed. In this scenario, the only decision variables are the stocking levels. Then we relax this model to allow the retailer to set retail prices and the manufacturers to set wholesale prices.

Using techniques presented in Heyman and Sobel (1984, Chapter 9), it can be shown that the infinite-horizon average profit-maximization problems for the manufacturer and the retailer can be studied using an equivalent single-period model. Thus, we develop and analyze an appropriate single-period model and will drop time subscript \( t \) in the rest of the paper. Table 1 contains the definition of model variables.

For given values of stocking levels \( q_i \) and demands \( d_i \), we get the following expressions for sales, inventory, and shortage for each brand for four possible cases shown in Figure 1.

1. \( d_i \leq q_i, d_2 \leq q_2 \)
   - sales \(_i = d_i \)
   - inventory \(_i = (q_i - d_i) \)
   - shortage \(_i = 0 \)

2. \( d_i \leq q_i, d_2 > q_2 \)
   - sales \(_1 = (d_i + \min\{\alpha(d_2 - q_2), q_1 - d_i\}) \)
   - sales \(_2 = (d_2 - \min\{\alpha(d_2 - q_2), q_1 - d_i\}) \)
   - inventory \(_1 = \max\{0, q_1 - d_i - \alpha(d_2 - q_2)\} \)
   - inventory \(_2 = 0 \)
   - shortage \(_1 = 0 \), shortage \(_2 = d_2 - q_2 \)

3. \( d_i > q_i, d_2 \leq q_2 \)
   - sales \(_1 = (d_i - \min\{\alpha(d_i - q_1), q_2 - d_2\}) \)
   - sales \(_2 = (d_2 - \min\{\alpha(d_i - q_1), q_2 - d_2\}) \)
   - inventory \(_1 = 0 \), inventory \(_2 = \max\{0, q_2 - d_2 - \alpha(d_i - q_1)\} \)
   - shortage \(_1 = d_i - q_1 \), shortage \(_2 = 0 \)

4. \( d_i > q_i, d_2 > q_2 \)
   - sales \(_1 = d_i \)
   - sales \(_2 = d_2 \)

\( \text{inventory}_1 = 0 \), \( \text{inventory}_2 = 0 \)

\( \text{shortage}_1 = d_1 - q_1 \), \( \text{shortage}_2 = d_2 - q_2 \).

Define

\[
j = 3 - i \quad A_i = \frac{q_i - d_i}{\alpha} \quad F(y) = \int_0^y f(x) \, dx.
\]

Noting that

\[
\min\{\alpha(d_i - q_i), q_i - d_i\} = \frac{\alpha(d_i - q_i)}{\alpha} \quad \text{if} \quad d_i \leq q_i - \alpha(d_i - q_i) \]

\[
q_i - d_i \quad \text{if} \quad d_i > q_i - \alpha(d_i - q_i),
\]

\[
\max\{0, q_i - d_i - \alpha(d_i - q_i)\} = \frac{q_i - d_i - \alpha(d_i - q_i)}{\alpha} \quad \text{if} \quad d_i \leq q_i - \alpha(d_i - q_i) \]

\[
0 \quad \text{if} \quad d_i > q_i - \alpha(d_i - q_i).
\]

### Figure 1: The Four Possible Regions for Demand

- **(i)** \( d_i \leq q_i \)
- **(ii)** \( d_i > q_i, d_2 \leq q_2 \)
- **(iii)** \( d_i > q_i, d_2 > q_2 \)
- **(iv)** \( d_1 \leq q_1, d_2 \leq q_2 \)

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\( \text{\textsuperscript{6}} \) See also Netessine et al. (2001) for an application of Heyman and Sobel’s technique under brand substitution.
Table 2

<table>
<thead>
<tr>
<th>Important Partial Derivatives</th>
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<tbody>
<tr>
<td>( \partial S_i / \partial q_i = \int_{a_i}^{A_i + \beta q_i} \int_{a_i}^{A_i + \beta q_i} f(x_i) f(x_j) , dx_i , dx_j )</td>
</tr>
<tr>
<td>( \partial S_r / \partial q_r = \int_{a_r}^{A_r + \beta q_r} \int_{a_r}^{A_r + \beta q_r} f(x_i) f(x_j) , dx_i , dx_j )</td>
</tr>
<tr>
<td>( \partial I_i / \partial q_i = \int_{a_i}^{A_i + \beta q_i} \int_{a_i}^{A_i + \beta q_i} \beta f(x_i) f(x_j) , dx_i , dx_j )</td>
</tr>
<tr>
<td>( \partial I_r / \partial q_r = \int_{a_r}^{A_r + \beta q_r} \int_{a_r}^{A_r + \beta q_r} \beta f(x_i) f(x_j) , dx_i , dx_j )</td>
</tr>
<tr>
<td>( \partial S_h / \partial q_h = -f(a_h + \beta q_h) f(x_i) )</td>
</tr>
<tr>
<td>( \partial S_l / \partial q_l = 0 )</td>
</tr>
<tr>
<td>( \partial S_r / \partial q_r = (1-\delta S_r / \partial q_r ) )</td>
</tr>
</tbody>
</table>

We can now compute

\[ S_i = E[d_i] + \int_{a_i}^{A_i + \beta q_i} \int_{a_i}^{A_i + \beta q_i} \alpha d_i q_i f(x_i) f(x_j) \, dx_i \, dx_j + \int_{a_i}^{A_i + \beta q_i} \int_{a_i}^{A_i + \beta q_i} (q_i - d_i) f(x_i) f(x_j) \, dx_i \, dx_j - \int_{a_i}^{a_i + \beta q_i} \int_{a_i}^{a_i + \beta q_i} \alpha d_i q_i f(x_i) f(x_j) \, dx_i \, dx_j + \int_{a_i}^{a_i + \beta q_i} \int_{a_i}^{a_i + \beta q_i} (q_i - d_i) f(x_i) f(x_j) \, dx_i \, dx_j \quad (3.1) \]

\[ I_i = \int_{a_i}^{A_i + \beta q_i} \int_{a_i}^{A_i + \beta q_i} (q_i - d_i) f(x_i) f(x_j) \, dx_i \, dx_j - \int_{a_i}^{a_i + \beta q_i} \int_{a_i}^{a_i + \beta q_i} \alpha d_i q_i f(x_i) f(x_j) \, dx_i \, dx_j \quad (3.2) \]

\[ S_h = \int_{a_h + \beta q_h}^{A_h + \beta q_h} (d_h - q_h) f(x_i) \, dx_i \quad (3.3) \]

We use the partial derivatives of the above quantities, shown in Table 2, in our analysis. We consider two scenarios. In the first scenario, we assume that all prices, both wholesale and retail, are exogenously fixed. In the second scenario, we allow manufacturers to set wholesale prices and the retailer to set retail prices. When prices are exogenous, brand substitution affects profits only through stocking levels. When prices are endogenously determined, brand substitution affects profits through stocking levels as well as prices.

4. Exogenous Wholesale and Retail Prices

We compute the optimal stocking levels and profits under VMI and RMI and compare the results to derive how VMI and RMI affect these quantities. We also analyze the effect of degree of substitution, \( \alpha \), on the results. We assume symmetric prices at the wholesale and retail levels in order to compare VMI and RMI directly. That is, we assume that

\[ p_1 = p_2 = p \quad \text{and} \quad w_1 = w_2 = w. \]

4.1. Analysis of RMI

Under RMI, the retailer’s profit is given by

\[ \pi_R = \sum_{i=1}^{2} ((p-w) S_i - h I_i - s S_h) \quad (4.1) \]

Manufacturer \( i \)'s profit is given by

\[ \pi_i = (w-c) S_i - S_h \quad (4.2) \]

The retailer determines \( q_i^{RMI}, i \in \{1, 2\}, \) by solving

\[ \frac{\partial \pi_R}{\partial q_i} = C_{RMI} = (p-w) \left( \frac{\partial S_i}{\partial q_i} + \frac{\partial S_h}{\partial q_h} \right) - \left( \frac{\partial I_i}{\partial q_i} + \frac{\partial I_h}{\partial q_h} \right) - s \frac{\partial S_h}{\partial q_h} = 0. \quad (4.3) \]

Note that when \( \alpha = 0, \partial I_i / \partial q_i = 0, \) implying that Equation (4.3) reduces to the standard newsvendor condition for the stocking level. Since the manufacturers and brands are symmetric in all respects, \( q_1^{RMI} = q_2^{RMI} = q^{RMI} \). The effect of brand substitution parameter \( \alpha \) on \( q^{RMI} \) is unclear under RMI. From implicit differentiation of Equation (4.3), we get

\[ \frac{\partial q_i^{RMI}}{\partial \alpha} = -\frac{\partial G^{RMI}_R}{\partial \alpha} = \left( \frac{\partial^2 \pi_R}{\partial q_i^2} \right)^{-1} \left[ -h \left( \int_{0}^{q_i+bq_i} \frac{q_i-x_i+\beta p}{\alpha^2} f(A_i+bq_i) f(x_i) \, dx_i \right) + \int_{0}^{q_i+bq_i} \frac{q_i-x_i+\beta p}{\alpha} f(A_i+bq_i) f(x_i) \, dx_i \right. \right. \]

\[ \left. - \int_{0}^{q_i+bq_i} \frac{A_i+\beta p}{\alpha} f(x_i) f(x_i) \, dx_i \right]_{q_i=\eta_1=\eta_2=RMI}. \]

While the denominator \( \left( \frac{\partial^2 \pi_R}{\partial q_i^2} \right)_{q_i=\eta_1=\eta_2=RMI} < 0, \) we cannot sign the numerator unambiguously. Still, the effect of \( \alpha \) on retailer profit can be stated as the following result.

**Lemma 1.** When prices are set exogenously, under RMI, retailer’s profit is increasing in \( \alpha; \) i.e., \( \partial \pi_R^{RMI} / \partial \alpha > 0. \)
PROOF. Let the optimal stocking level when \( \alpha = \alpha_1 \) be \( q^{RMI}(\alpha_1) \). Let the optimal profit of the retailer when \( \alpha = \alpha_1 \) be \( \pi^{RMI}(\alpha_1, q^{RMI}(\alpha_1)) \). Consider \( \alpha_2 > \alpha_1 \). Since
\[
\frac{\partial \pi^{RMI}}{\partial \alpha} = \int_{0}^{\alpha_2} \int_{0}^{B_1 + \beta p} (q_i - x_i + \beta p) f(x_i) f(x_i) dx_i dx_2 \geq 0,
\]
\[
\pi^{RMI}(\alpha_2, q^{RMI}(\alpha_1)) \geq \pi^{RMI}(\alpha_1, q^{RMI}(\alpha_1)).
\]
Consequently, \( \pi^{RMI}(\alpha_2, q^*(\alpha_2)) \geq \pi^{RMI}(\alpha_2, q^*(\alpha_1)) \geq \pi^{RMI}(\alpha_1, q^*(\alpha_1)). \]

Lemma 1 is intuitive and can be explained as follows. A higher degree of substitution reduces the retailer’s inventory level because more customers buy any brand that is in stock. The retailer can realize a higher profit when \( \alpha \) increases even if he does not adjust his stocking level. Consequently, the retailer can never be worse off when \( \alpha \) increases.

4.2. Analysis of VMI
Under VMI, the retailer’s profit is given by
\[
\pi_R = \sum_{i=1}^{2} (p - w) S_i - s S h_i. \tag{4.4}
\]
Manufacturer \( i \)'s profit is given by
\[
\pi_i = (w - c) S_i - h I_i - s S h_i. \tag{4.5}
\]
Manufacturer \( i \) determines \( q^{VMI}_i \) by solving
\[
\frac{\partial \pi_i}{\partial q_i} = \frac{\partial \pi^{VMI}_i}{\partial \alpha} = (w - c) \frac{\partial \pi_i}{\partial q_i} - h \frac{\partial I_i}{\partial q_i} - s \frac{\partial h_i}{\partial q_i} = 0. \tag{4.6}
\]
When \( \alpha = 0 \), \( \partial S_i / \partial q_i = 0 \). Therefore, the first-order condition for the stocking level under VMI (i.e., Equation (4.6)) becomes identical to the first-order condition under RMI (i.e., Equation (4.3)) when the brands are not substitutable. Consequently, when \( \alpha = 0 \), the stocking levels are the same, which is equal to the standard newsvendor stocking level, irrespective of who makes the decision. Again, because the manufacturers and brands are symmetric in all respects, \( q^{VMI}_1 = q^{VMI}_2 = q^{VMI} \). The effects of \( \alpha \) on the stocking levels and profits are stated as the following result.

Lemma 2. When prices are set exogenously, under VMI:

(i) The stocking level is increasing in \( \alpha \); i.e., \( \partial q^{VMI}/\partial \alpha > 0 \).

(ii) The retailer’s profit is increasing in \( \alpha \); i.e., \( \partial \pi^{VMI}_R/\partial \alpha > 0 \).

PROOF. (i) The result follows directly from implicit differentiation of (4.6).
\[
\frac{\partial \pi^{VMI}_R}{\partial \alpha} = -\frac{\partial \pi^{VMI}_R}{\partial q} \frac{\partial q}{\partial \alpha} = -\frac{\partial \pi^{VMI}_R}{\partial q} \frac{\partial q}{\partial \alpha} \geq 0.
\]

(ii) \( \frac{\partial \pi^{VMI}_R}{\partial \alpha} = \frac{\partial \pi^{VMI}_R}{\partial q} \frac{\partial q}{\partial \alpha} \geq 0 \).

The reasons that manufacturers stock more for a higher \( \alpha \) can be explained as follows. Under VMI, manufacturers face two opportunity costs while deciding on their stocking levels. The first is the profit lost by a manufacturer if his brand goes out of stock and some of his customers buy the competing brand. The second is the profit a manufacturer can realize if the competing brand goes out of stock and some of the competitor’s customers buy his brand. Unlike RMI, in which the retailer’s trade-off in determining the stocking level depends only on holding costs and shortage costs, the manufacturers under VMI face this additional component, opportunity costs, to take into consideration. Consequently, under VMI, the “real” shortage cost can be substantially higher than \( s \). As the degree of substitution increases, the potential spillover demand as well as lost sales also increases, and the opportunity cost increases. Manufacturers respond to this higher shortage cost by increasing their stocking levels. The retailer clearly benefits from higher stocking levels because higher stocking levels reduce the retailer’s shortage costs.

4.2.1. Comparison of Results Under RMI and VMI. We compare the stocking levels and retailer’s profits under RMI and VMI in the following results.

Lemma 3. When prices are exogenous, (i) if the brands are not substitutable, the stocking levels are identical under RMI and VMI; i.e., if \( \alpha = 0 \), then \( q^{VMI} = q^{RMI} \); (ii) if the brands are substitutable, for any degree of substitution, the stocking level under VMI is not lower than under RMI; i.e., for any \( \alpha > 0 \), \( q^{VMI} \geq q^{RMI} \).
PROOF.
(i) When $\alpha = 0$, (4.4) and (4.10) become identical equations. Consequently, $q^{VMI} = q^{RMI}$.
(ii) We know from (4.4) and (4.10) that
\[
- h \left( \frac{\partial l_i}{\partial q_i} + \frac{\partial l_i}{\partial q_i} \right)_{q=q^{RMI}} - s \left( \frac{\partial h_i}{\partial q_i} \right)_{q=q^{RMI}} = 0
\]

\[
\Rightarrow - h \left( \frac{\partial l_i}{\partial q_i} \right)_{q=q^{RMI}} - s \left( \frac{\partial h_i}{\partial q_i} \right)_{q=q^{VMI}} = h \left( \frac{\partial l_i}{\partial q_i} \right)_{q=q^{RMI}}
\]

\[
(w-c)\left( \frac{\partial S_i}{\partial q_i} \right)_{q=q^{VMI}} - h \left( \frac{\partial l_i}{\partial q_i} \right)_{q=q^{VMI}} - s \left( \frac{\partial h_i}{\partial q_i} \right)_{q=q^{VMI}} = 0
\]

\[
\Rightarrow - h \left( \frac{\partial l_i}{\partial q_i} \right)_{q=q^{VMI}} = s \left( \frac{\partial h_i}{\partial q_i} \right)_{q=q^{VMI}}
\]

\[
= -(w-c) \left( \frac{\partial S_i}{\partial q_i} \right)_{q=q^{VMI}}.
\]

Since $\partial l_i / \partial q_i \geq 0$, $\partial S_i / \partial q_i \geq 0$, and $- h(\partial l_i / \partial q_i) - s(\partial h_i / \partial q_i)$ is a decreasing function of $q_i$, hence $q^{VMI} \geq q^{RMI}$. \qed

Lemma 3 shows that when there is no brand substitution, the stocking level remains the same irrespective of whether the retailer or the manufacturers set the stocking levels. Thus, expected sales, inventories as well as shortage, will be equal in both RMI and VMI. However, the retailer’s (manufacturers’) profit will be higher (lower) in VMI because the inventory holding cost is shifted to the manufacturers in VMI. Now our principal theorem about the effect of RMI and VMI on retailer profit follows.

**Theorem 1.** When prices are exogenous, the retailer’s profit under VMI is greater than or equal to the retailer’s profit under RMI, i.e., $\pi_{R}^{VMI} \geq \pi_{R}^{RMI}$.

**Proof.** From Lemma 3, $q^{VMI} \geq q^{RMI}$.

\[
\pi_{R}^{VMI} - \pi_{R}^{RMI} = (p-w)(S_{1}^{VMI} + S_{2}^{VMI} - S_{1}^{RMI} - S_{2}^{RMI}) + h(S_{1}^{RMI} + S_{2}^{RMI}) - s(S_{1}^{VMI} + S_{2}^{VMI} - S_{1}^{RMI} - S_{2}^{RMI}).
\]

Since $S_{1}^{VMI} + S_{2}^{VMI} = S_{1}^{RMI} + S_{2}^{RMI}$ when $q_{i}^{VMI} = q_{i}^{RMI}$ and $q_{i}^{VMI} = q_{i}^{RMI}$, $h(S_{1}^{RMI} + S_{2}^{RMI}) \geq 0$. Since $\partial h_i / \partial q_i \leq 0$, $S_{1}^{VMI} + S_{2}^{VMI} \leq S_{1}^{RMI} + S_{2}^{RMI}$. So, $\pi_{R}^{VMI} - \pi_{R}^{RMI} \geq 0$. \qed

The following observations are worth noting from the above result. The higher profit that the retailer realizes under VMI compared to RMI is not solely because the inventory holding cost is transferred to the manufacturer under VMI. The retailer also realizes a lower shortage cost under VMI than RMI. From the proof of Lemma 3, we can also observe that the stocking levels in VMI increase with an increase in the wholesale price. Thus, the retailer realizes an even higher profit under VMI when $w$ is higher. The manufacturers incur the inventory holding costs in VMI, which they do not in RMI. When the brands are not substitutable, the manufacturers’ shortage costs remain the same in both VMI and RMI, and so the manufacturers are worse off in VMI in than RMI. When the brands are substitutable, the manufacturers also realize lower shortage costs because of higher stocking levels. Consequently, it is not clear whether the manufacturers will always be worse off under VMI compared to RMI.

In summary, when wholesale and retail prices are exogenously fixed, the retailer can exploit the competition between manufacturers because of brand substitution by delegating the stocking-level decisions to manufacturers. When manufacturers make stocking-level decisions, brand substitution makes them compete on quality, which increases stocking levels. The higher stocking levels reduce retailer’s shortages. When the retailer makes the stocking-level decisions, the manufacturers do not engage in quantity competition because the retailer makes the decisions based on the aggregate demand.

5. **Endogenous Prices**

In this section, we relax our model from §4 and endogenize retail and wholesale prices. The sequence of actions in this case is as follows. In the first stage, the manufacturers set wholesale prices simultaneously. Then, in the second stage, the retailer sets retail prices. In the third stage, depending on whether it is RMI or VMI, either the retailer or the manufacturers set stocking levels. Finally, the demands are realized in the last stage. During any stage of the game, the players know the decisions made during the earlier stages. We use the backward induction procedure and solve from the last stage.

5.1. **RMI**

The retailer’s profit under RMI is again given by Equation (4.1), and manufacturer $i$’s profit under RMI is given by Equation (4.5). In the case of RMI, the retailer’s information set does not change between the second and third stages. In this case, as shown by Petruzzi and Dada (1999), the optimal stocking levels and prices are identical whether they are solved simultaneously or sequentially. Thus, the retailer determines $q_i^*$ and $p_i^*$, $i \in \{1, 2\}$, by solving

\[
\frac{\partial \pi_{R}^{i}}{\partial q_i} = G_{RMI}^{i},
\]

\[= (p_i - w_i) \frac{\partial S_i}{\partial p_i} + S_i + (p_i - w_i) \frac{\partial S_i}{\partial p_i},
\]

\[- h \left( \frac{\partial l_i}{\partial q_i} + \frac{\partial l_i}{\partial q_i} \right) - s \left( \frac{\partial h_i}{\partial q_i} + \frac{\partial h_i}{\partial q_i} \right).
\]
\[ (p_i - w_i - p_j + w_j) \frac{\partial s_i}{\partial q_i} + s_i - \beta(p_i - w) \]
\[ - h(\frac{\partial l_j}{\partial p_i} + \frac{\partial l_j}{\partial q_i}) - s \frac{\partial s_i}{\partial q_i} \]
\[ = -\beta(p_i - w_i - p_j + w_j) \left( 1 - \frac{\partial s_i}{\partial q_i} \right) + s_i - \beta(p_i - w) \]
\[ - h \beta \left( \frac{\partial l_j}{\partial q_i} + \frac{\partial l_j}{\partial q_i} \right) - s \beta \frac{\partial s_i}{\partial q_i} = 0 \]  
(5.1)

\[ \frac{\partial \pi_{RMI}}{\partial q_i} = G_{\pi_{RMI}}(q_i, q_j, p_i, p_j) \]
\[ = (p_i - w_i) \frac{\partial s_i}{\partial q_i} + (p_j - w_j) \frac{\partial s_j}{\partial q_i} \]
\[ - h \left( \frac{\partial l_i}{\partial q_j} + \frac{\partial l_j}{\partial q_j} \right) - s \left( \frac{\partial s_i}{\partial q_i} + \frac{\partial s_j}{\partial q_i} \right) \]
\[ = (p_i - w_i - p_j + w_j) \frac{\partial s_i}{\partial q_i} - h \left( \frac{\partial l_i}{\partial q_i} + \frac{\partial l_j}{\partial q_i} \right) \]
\[ - s \frac{\partial s_i}{\partial q_i} = 0. \]  
(5.2)

By substituting (5.2) in (5.1), we get
\[ (p_i - w_i) \frac{\partial s_i}{\partial q_i} + (p_j - w_j) \frac{\partial s_j}{\partial q_i} \]
\[ = s_i - \beta(p_i - w) = 0, \]  
(5.3)

which yields the following optimal prices.
\[ p_i^* = \frac{1}{2} \left( \frac{a}{\beta} + w_i \right). \]  
(5.4)

Note that \( p_i^* \) is independent of \( \alpha \) and is identical to the price the retailer would have set if the demand of each brand was equal to the mean demand \( a \). Thus, \( q_i^* \) satisfies the following equation
\[ \frac{(w_j - w_i)}{2} \frac{\partial s_j}{\partial q_i} - h \left( \frac{\partial l_i}{\partial q_i} + \frac{\partial l_j}{\partial q_i} \right) - s \frac{\partial s_i}{\partial q_i} = 0. \]  
(5.5)

Now make the transformation \( z_i = q_i + \beta p_i \) and substitute the partial derivatives from Table 2 in Equation (5.5). We get
\[ \frac{(w_j - w_i)}{2} \int_{z_i + \beta(p_j - w)}^{\infty} f(x_i) f(x_j) dx_i dx_j \]
\[ - h \left( \int_{0}^{\infty} \int_{0}^{\infty} f(x_i) f(x_j) dx_i dx_j \right) \]
\[ + s \int_{z_i}^{\infty} f(x_i) dx_i = 0. \]  
(5.6)

Note that Equation (5.6) is independent of \( p_i \) and \( p_j \). If \( z_i^* \) is the solution to Equation (5.6), then \( q_i^* = z_i^* - \beta p_i^* \).

The previous observation that the retailer sets retail prices based only on the expected demands of brands, combined with Equation (5.6), implies that the degree of substitution affects only the stocking-level decisions. That is, even when the retailer is allowed to adjust retail prices, he adjusts only the stocking levels in response to changes in the degree of substitution as he does when retail price is exogenously set.

In the first stage, manufacturer \( i \) determines \( w_i \) by solving
\[ \frac{\partial \pi_i}{\partial w_i} = G_{\pi_{RMI}}(w_i, q_i, q_j, p_i, p_j) \]
\[ = (w_i - c) \frac{\partial s_i}{\partial w_i} + s_i \left( \frac{\partial s_i}{\partial w_i} \right) \]
\[ = (w_i - c) \left( \frac{1 \frac{\partial s_i}{\partial p_i} + \frac{\partial s_i}{\partial q_i}}{2 \frac{\partial p_i}{\partial q_i}} \right) + s_i \]
\[ - s \left( \frac{1 \frac{\partial s_i}{\partial p_i} + \frac{\partial s_i}{\partial q_i}}{2 \frac{\partial p_i}{\partial q_i}} \right) \]
\[ = (w_i - c) \left( \frac{-\beta}{2} \right) + s_i = 0. \]  
(5.7)

The solution to Equation (5.7) yields
\[ w_{i}^{\pi_{RMI}} = w_{i}^{\pi_{RMI}} = w_{i}^{\pi_{RMI}} = \frac{a + \beta c}{2\beta}. \]  
(5.8)

The effects of \( \alpha \) on stocking levels and retailer profit are given by the following result.

**Lemma 4.** When prices are not exogenous, in RMI,
(i) the retail and wholesale prices are independent of \( \alpha \),
(ii) the retailer’s profit is increasing in \( \alpha \), i.e., \( \frac{\partial \pi_{RMI}}{\partial \alpha} > 0 \).

**Proof.**
(i) Follows from (5.4) and (5.8).
(ii) The proof for this result follows the same steps as those given in the proof for Lemma 1. \( \square \)

### 5.2. VMI
The retailer’s profit under VMI is given by Equation (4.4), and manufacturer \( i \)’s profit under RMI is given by Equation (4.5). We determine the equilibrium prices and stocking levels using backward induction by solving for manufacturers’ stocking levels given prices first, followed by optimal retail prices, and then optimal wholesale prices.

Manufacturer \( i \) determines \( q_i^{\pi_{VMI}} \) by solving
\[ \frac{\partial \pi_i}{\partial q_i} = G_{\pi_{VMI}}(w_i, q_i, q_j, p_i, p_j) \]
\[ = (w_i - c) \frac{\partial s_i}{\partial w_i} - h \left( \frac{\partial l_i}{\partial q_i} - s \frac{\partial s_i}{\partial q_i} \right) = 0. \]  
(5.9)

In order to solve (5.9), we assume that both manufacturers conjecture that the other manufacturer’s stocking level is a monotone function of its own price of the
form \( Q - \beta p_i \) for manufacturer \( i \), and we shall show that this conjecture is correct in the equilibrium. That is, if manufacturer \( i \) uses \( q_i^* = Q - \beta p_i \), the unique best response for manufacturer \( j \) is to use \( q_j^* = Q - \beta p_j \). To see this, substitute \( q_j = Q - \beta p_j \) in Equation (5.9). Then, Equation (5.9) becomes

\[
\begin{align*}
(w - c) \int Q^{-\alpha} f(x_i) dx_i & = (w - c) \int_{Q^{-\alpha_{1}}(Q^{-\alpha_{1}} - \beta)} f(x_i) dx_i \\
- h \int Q^{-\alpha_{2}} f(x_i) dx_i & = -(1) \int_{Q^{-\alpha_{2}}(Q^{-\alpha_{2}} - \beta)} f(x_i) dx_i \\
+ s \int_{Q^{-\alpha_{3}}} f(x_i) dx_i & = 0.
\end{align*}
\]

Again, let \( z_i = q_i + \beta p_i \) and \( z_i^* \) be the solution to Equation (5.10), then \( q_i^* = z_i^* - \beta p_i \). Note that \( z_i^* \) is independent of \( p_i \). Thus, the equilibrium stocking levels in Stage 3 of the game is given by the following:

\[
q_i^* = Q - \beta p_i,
\]

where \( Q \) solves the equation

\[
(w - c) \int Q^{-\alpha} f(x_i) dx_i = (w - c) \int_{Q^{-\alpha_{1}}(Q^{-\alpha_{1}} - \beta)} f(x_i) dx_i \\
- h \int Q^{-\alpha_{2}} f(x_i) dx_i = -(1) \int_{Q^{-\alpha_{2}}(Q^{-\alpha_{2}} - \beta)} f(x_i) dx_i \\
+ s \int_{Q} f(x_i) dx_i = 0.
\]

We see that

\[
\frac{\partial q_i^*}{\partial p_i} = -\beta \quad \text{and} \quad \frac{\partial q_i^*}{\partial q_i} = 0.
\]

The retailer determines \( p_i^* \), \( i \in \{1, 2\} \), by solving

\[
\frac{\partial \pi_v}{\partial p_i} = G_{p}^{\text{VMI}} = (p_i - w) \left( \frac{\partial S_i}{\partial p_i} + \frac{\partial S_i}{\partial q_i} \right) + S_i \\
+ (p_i - w) \left( \frac{\partial S_i}{\partial p_i} + \frac{\partial S_i}{\partial q_i} \right) + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i^*}{\partial q_i} \\
- S_i \left( \frac{\partial S_i}{\partial p_i} + \frac{\partial S_i}{\partial q_i} \right) + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i^*}{\partial q_i} \\
+ \frac{\partial S_i}{\partial p_i} + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i^*}{\partial q_i} + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i^*}{\partial q_i} \\
+ \frac{\partial S_i}{\partial p_i} + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i^*}{\partial q_i} + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i^*}{\partial q_i} \\
= S_i - \beta(p_i - w) = 0.
\]

Thus, the equilibrium prices are given by

\[
p_i^* = \frac{1}{2}(s + w).
\]

Now in the first stage, manufacturer \( i \) solves the following equation to determine \( w_i^* \).

\[
\frac{\partial \pi_i}{\partial w_i} = G_{\text{VMI}}^{\text{VMI}} = \frac{(w_i - c) \frac{\partial S_i}{\partial w_i} - h \frac{\partial S_i}{\partial w_i}}{2} - s \left( \frac{\partial S_i}{\partial w_i} \right) = \left( w_i - c \right) \left( \frac{1}{2} \frac{\partial S_i}{\partial w_i} + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i}{\partial w_i} \right) + s_i \\
- h \left( \frac{1}{2} \frac{\partial S_i}{\partial w_i} + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i}{\partial w_i} \right) - s \left( \frac{1}{2} \frac{\partial S_i}{\partial w_i} + \frac{\partial S_i}{\partial q_i} \frac{\partial q_i}{\partial w_i} \right) \\
= (w_i - c) \left( \frac{-\beta}{2} \right) + s_i = 0.
\]

Solution to Equation (5.14) yields

\[
\omega_i^* = \omega_i^* = \omega_i^* = \frac{s + \beta c}{2\beta}.
\]

The following result shows the effect of brand sub-stitution on retail price, stocking level, and retailer profit in VMI.

**Lemma 5.** When prices are not exogenous in VMI, (i) the retail and wholesale prices are independent of \( \alpha \), (ii) the stocking level is increasing in \( \alpha \); i.e., \( \frac{\partial q_i^*}{\partial \alpha} \geq 0 \), (iii) the retailer's profit is increasing in \( \alpha \); i.e., \( \frac{\partial \pi_v^*}{\partial \alpha} \geq 0 \).

**Proof.**

(i) The proof follows directly from Equation (5.13) and Equation (5.15).

(ii) Implicit differentiation of Equation (5.11) shows that \( \frac{\partial q_i^*}{\partial \alpha} \geq 0 \).

(iii) \( \frac{\partial \pi_v^*}{\partial \alpha} = \left( \frac{\partial \pi_v^*}{\partial q_i^*} \right) \frac{\partial q_i^*}{\partial \alpha} = 2sf(\omega_i^* + \beta p_i)(\omega_i^* + \beta p_i) \geq 0 \)

5.3. Comparison of Results Under RMI and VMI

In order to compare the retailer profit in RMI and VMI, we show the following result first.

**Lemma 6.** When prices are not exogenous, (i) the retail and wholesale prices are identical in VMI and RMI; (ii) if the brands are not substitutable, the stocking levels are identical under RMI and VMI; i.e., if \( \alpha = 0 \), then \( q_i^* \geq q_i^* \); (iii) if the brands are substitutable, for any degree of substitution, the stocking level under VMI is not lower than under RMI; i.e., for any \( \alpha > 0 \), \( q_i^* \geq q_i^* \).

**Proof.**

(i) The proof follows from the price expressions.

(ii) The proof follows from Equation (5.5) and Equation (5.10) become identical equations. Consequently, \( q_i^* = q_i^* \).
The proof is straightforward when we compare Equation (5.5) with Equation (5.9). We know from Equation (5.5) and Equation (5.9) that

\[-h \left( \frac{\partial l_i}{\partial q_i} + \frac{\partial l_i}{\partial q_i} \right)_{q_i = p^{\text{RMI}}} - s \left( \frac{\partial S_i}{\partial q_i} \right)_{q_i = p^{\text{RMI}}} = 0\]

\[\Rightarrow -h \left( \frac{\partial l_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} - s \left( \frac{\partial S_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} = h \left( \frac{\partial l_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} \]

\[(w-c) \left( \frac{\partial S_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} - h \left( \frac{\partial l_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} - s \left( \frac{\partial S_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} = 0\]

\[\Rightarrow -h \left( \frac{\partial l_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} - s \left( \frac{\partial S_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} = 0\]

\[\Rightarrow (w-c) \left( \frac{\partial S_i}{\partial q_i} \right)_{q_i = p^{\text{VMI}}} = 0\]

Since

\[\frac{\partial l_i}{\partial q_i} > 0, \quad \frac{\partial S_i}{\partial q_i} > 0,\]

and

\[-h \left( \frac{\partial l_i}{\partial q_i} \right) - s \left( \frac{\partial S_i}{\partial q_i} \right)\]

is a decreasing function of \(q_i\), it follows that \(p^{\text{VMI}} \geq p^{\text{RMI}}\) \(\square\)

Now our principal theorem about the effects of RMI and VMI on retailer profit follows.

**Theorem 2.** When prices are not exogenous, the retailer's profit under VMI is at least as large as the retailer's profit under RMI; i.e., \(\pi^{\text{VMI}} \geq \pi^{\text{RMI}}\).

**Proof.** The proof is similar to the proof for Theorem 1. \(\square\)

The results of this section show that even when the manufacturers and the retailer are allowed to set prices, the principal result, that VMI benefits the retailer, holds. The prices do not depend on who, retailer or manufacturer, manages the inventory. However, the stocking levels are higher under VMI. Thus, VMI results in lower shortages while keeping the prices and profit margins the same for the retailer.

In essence, the higher profit that the retailer realizes under VMI is solely because of the quantity competition that is absent in RMI, but is restored by VMI. When there is no substitution, both VMI and RMI yield the same profits. The result that retailer profit increases with higher degree of substitution implies that higher substitution intensifies the quantity competition between manufacturers.

As in the exogenous price case, we are unable to show unambiguously whether the shift from RMI to VMI benefits or hurts the manufacturers. Another interesting question to investigate is how the shift affects the system profit, defined as the sum of retailer profit and manufacturer profits. We find that the system profit under VMI may be higher or lower compared to RMI. The system profit is given by

\[\pi = \sum_{i=1}^{2} (p - c)S_i - hL_i - 2sS_i.\] (5.16)

The maximum system profit is obtained when the stocking levels satisfy the following first-order conditions.

\[-h \left( \frac{\partial l_i}{\partial q_i} + \frac{\partial l_i}{\partial q_i} \right) - 2s \left( \frac{\partial S_i}{\partial q_i} \right) = 0, \quad i \in \{1, 2\}.\] (5.17)

In order to show that the system profit under VMI may be lower than under RMI, we consider the case when \(b > 0\) and \(c = 0\). We can readily show that, under RMI, in both exogenous and endogenous price cases, the stocking level for each brand will be the lower support of the demand distribution. From (5.17), we can also observe that this stocking level also maximizes system profit. However, under VMI, Equation (4.6) and Lemma 3 in the exogenous price case, and Equation (5.9) and Lemma 6 in the endogenous price case, show that the stocking levels will be higher than under RMI; i.e., higher than the optimal stocking levels that maximize system profit. Consequently, when \(c = 0\), we can conclude that the system profit is lower under VMI than under RMI. We are unable to identify meaningful conditions on parameters for which the system profit under VMI will be higher than under RMI. However, numerical analysis presented in Table 3 in §6 shows that system profit can indeed be higher under VMI than under RMI.

Further analysis of VMI and RMI for the general demand distributions is difficult. In the next section, we restrict our analysis to uniformly distributed demands in order to gain further insights into the effect of brand substitution, RMI, and VMI on retailer as well as manufacturer profits.

### 6. An Illustrative Example with Uniformly Distributed Demands

Our analysis so far has not made any restrictions on the demand distributions. For the general model, we were unable to derive the effect of degree of brand substitution on manufacturer profits. Also, as we noted in §4, we could not show how \(\alpha\) affects the stocking level in RMI. The purpose of this section is to provide additional insights about these questions.

We restricted our focus to uniformly distributed demands. Specifically, \(X_i\) was assumed to follow \(U(\alpha, \alpha + 2c)\). The stocking levels under RMI and VMI

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### Table 3: Numerical Solutions for Uniformly Distributed Demands

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<th>Stocking level</th>
<th>RMI</th>
<th>VMI</th>
<th>RMI</th>
<th>VMI</th>
<th>RMI</th>
<th>VMI</th>
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<th>RMI</th>
<th>VMI</th>
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can be computed to be the following:

\[
q_{\text{RMI}}^{*} = \frac{(a - \beta c) + e(4 + a(c + 3a))}{2\alpha(3 + \alpha)} + \frac{e(4s - 4s^2 + 2hs(1 - \alpha) + (1 + \alpha)(1 + 3\alpha))^{1/2}}{2h\alpha(3 + \alpha)}. \tag{6.1}
\]

\[
q_{\text{VMI}}^{*} = \frac{(a - \beta c)}{4} + \frac{2e(h + s + h\alpha)}{\alpha(h + c(1 + \alpha) - w(1 + \alpha))} + \frac{1}{\alpha(h + c(1 + \alpha) - w(1 + \alpha))} \cdot 2e((c - w)\alpha^2 - h^2(1 + 2\alpha) + (s - c\alpha + w\alpha^2) + 2h(s + (c - w)\alpha^2))^{1/2}. \tag{6.2}
\]

The sensitivity of stocking level, inventory, shortage, and profits with respect to \( \alpha \) are too complex to provide meaningful insights. Consequently, we report numerical solutions to make our illustration (Table 3). We used the following parameter values for our illustration: \( a = 1,000; e = 100; h = 5; s = 5; c = 25; \beta = 25. \)

The numerical results confirm our theoretical results. The following additional insights are worth noting from the above results.

(i) The stocking level under RMI may increase or decrease as the degree of substitution increases. In the above example, the stocking level is decreasing in \( \alpha \) for \( \alpha \leq 0.5 \) and increasing in \( \alpha \) for \( \alpha > 0.5 \). Thus, a higher level of pooling of demands because of higher degree of brand substitution does not always decrease the stocking level.

(ii) The manufacturers’ profit under RMI may also increase or decrease as the degree of substitution increases. Under RMI, the manufacturers’ profit follows the same behavior as that of the stocking level. This is because the manufacturers’ profit depends on the shortage cost, which is directly dependent on the stocking level. A higher (lower) stocking level reduces (increases) the shortage cost, which, in turn, increases (decreases) the manufacturers’ profit.

(iii) The manufacturers’ profit under VMI may increase or decrease as the degree of substitution increases. Under VMI, the manufacturers’ profit depends on the inventory holding costs and shortage costs. While the manufacturers increase the stocking levels when the degree of substitution increases under VMI, the extent to which the higher stocking level increases inventory holding cost and decreases the shortage cost depends on the degree of substitution. However, numerical results show that manufacturers’ profit is lower under VMI than under RMI for all values of \( \alpha \).

(iv) The system profit is increasing in \( \alpha \) in both RMI and VMI. The system profit is higher under VMI than RMI.

### 7. Discussion and Conclusions

In this paper, we offered an alternative explanation not documented in prior literature as to why retailers might be interested in shifting the burden of managing retail inventory to manufacturers through the VMI mechanism. Prior research has documented the potential benefits VMI offered to retailers, such as transfer of monitoring and other overhead costs to manufacturers, and continuous replenishment of inventory. While these benefits may very well accrue to the retailer, we showed in this paper that retailers could exploit the competition between manufacturers of substitutable products better through VMI than through the traditional RMI. The increased competition between brands under VMI causes manufacturers to stock more than what the retailer would have stocked. Consequently, VMI, in addition to eliminating retailer’s holding cost, reduces retailer’s shortage costs as well and increases his profit. Higher degree of substitution results in higher retailer profit under VMI.

Our results suggest that retailers will push manufacturers to adopt VMI. Even though retailers delegate control over an important variable, i.e., stocking
level, that determines their profits to manufacturers under VMI, they are better off under VMI compared to RMI. As for the manufacturers, it is not clear why they should accept VMI, because it intensifies quantity competition among them. If a retailer has control over the manufacturers (for instance, a large retailer such as Wal-Mart may have control over manufacturers), then manufacturers may be forced to adopt VMI. In the EDI context, it is a documented fact that large buyers such as Wal-Mart and Chrysler were able to force their suppliers to adopt EDI even though suppliers resisted it. If a retailer cannot force manufacturers to adopt VMI, then it may have to provide appropriate incentives to manufacturers. Lee et al. (2000a) stated that manufacturers’ adoption of VMI is an incentive manufacturers may provide to retailers in return for sharing their demand and inventory information with manufacturers. Our results suggest that it is conceivable that demand information sharing by retailers could be the incentive retailers provide to manufacturers in return for adopting VMI.

It should be emphasized that we were unable to prove that manufacturers would always be worse off under VMI compared to RMI. In our model, the retailer and manufacturers incur the same holding cost. We made the assumption in order to isolate the effect of supply chain structure, VMI versus RMI, on brand competition and retailer profits. It is quite likely that manufacturers incur a smaller inventory holding cost than the retailer. If the manufacturers’ inventory holding cost is sufficiently low, then the manufacturers may also be better off under VMI than RMI. In such scenarios, the manufacturers will also be willing partners in the VMI program.

We presented a model in which (i) the demand of a brand depended only on its own retail price, and (ii) the shortages at the retailer after possible substitution were backordered. We analyzed several variations of this model and found qualitatively the same results. For example, in the first variation, we allowed the demand of a brand to depend not only on its own retail price, but also the price of the competing brand. Thus, we used the following demand function:

\[ d_i = X_i - \beta p_i + \gamma p_j, \quad i \in \{1, 2\}, \quad \beta, \gamma > 0, \quad \beta > \gamma, \]

The only change to our results was in the price expressions. Specifically, \( \beta \) was replaced by \( (\beta - \gamma) \) in the price expressions. This is intuitive because (a) the prices depend only on the expected demands, and (b) the stocking level is the sum of mean demand and safety stock that is independent of prices. Since the qualitative results of our analysis are affected only by the safety stock levels, the effect of prices on mean demands do not influence our results. In the second variation, we considered a lost-sale model in which the shortages after substitution are lost instead of backordered. This model is significantly more complex to analyze than the model with backorders. We were able to show analytically that our primary result that the retailer is better off under VMI than RMI holds for the case when (i) both retail and wholesale prices are set exogenously, and (ii) retail prices are endogenously determined, but the wholesale prices are set exogenously. The model with exogenous wholesale prices was intractable. However, numerical simulations showed that the results hold for this case also.

There are several extensions to our research. We assumed that there is a single retailer and two manufacturers of potentially substitutable products. Thus, competition exists only between manufacturers. A more general model might also include competition between two or more retailers. This will require us to model not only the intraretailer interbrand substitution that we modeled in this paper but also the interretailer intra-brand substitution. The latter substitution is the result of consumers searching for the same brand at a different store when the first retailer doesn’t have that brand in stock. The interaction of these substitution effects could potentially change the optimal channel structure. Currently, we are investigating such a model to generalize our results further.

Acknowledgments

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References


