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Impact of introducing make-to-order options in a make-to-stock environment

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Abstract

Firms engaged in consumer product sales often implement a strict make-to-stock approach, applying a single price to all customers. In such systems, customers can get the product at the given price upon availability on the shelf. However, consumers can often tolerate a delay between order placement and demand satisfaction under a price discount. Recognizing this phenomenon, a supplier may consider offering a menu of delivery-price options to consumers, where longer delay-time options imply lower prices. Demands from customers willing to wait provide advance demand information to the supplier. This paper studies strategies to exploit this additional information to improve profitability and service levels. Primarily assuming that delivery times are set exogenously, we determine optimal prices and stock levels under the new delayed demand satisfaction options. In addition, we develop analytical models to characterize the system performance gains under the new demand fulfillment option.

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1. Introduction

In consumer product sales environments a supplier often sets a single price for a good for all consumers. That is, any customer can get the product at the single price upon availability. Under this approach, customers are assumed homogenous in terms of reservation price and reaction to product availability. Such a strategy ignores key consumer-preference factors that may impact the supplier’s profitability. In marketing research and practice, it is generally believed that consumers can be segmented according to preferences on
key product features such as price, quality, and availability (Porter, 1980). Past economics literature also recognizes that consumers have different reservation prices for a common good (McConnell and Brue, 2001). In many contexts, consumers with lower reservation prices may be willing to accept delayed demand satisfaction, i.e., longer order lead times in exchange for a lower price. This is particularly true for items ordered on-line, since these customers naturally expect some delay between order placement and demand satisfaction.

Amazon.com, for example, recognizes such customer differences, and recently initiated a series of practices to cater to distinct needs. Its free-shipping policy is a successful example of such efforts. As noted in an article in the Asian Wall Street Journal on November 22, 2002, “… Amazon notices that, in exchange for free shipping (which leads to a lower unit price), customers typically agree to wait several extra days to receive their goods. Of course, they can choose to pay for faster shipping at a higher unit price…” Amazon thus provides a menu of delivery-time price alternatives for its customers. Lower unit prices may reduce total revenue; however, such negative effects can be countered by increased sales volumes. Perhaps more importantly, a high degree of delivery flexibility can help significantly reduce operations costs. The key driver is the increased reaction time that the supplier has to meet demands, which reduces the demand uncertainty the supplier faces. The longer delivery time allowed for customer orders provides firms such as Amazon with advance information that allows consolidating sales and more economically and reliably scheduling transportation. As a result, customers are satisfied and retained due to high service rates and enticing discount policies.

This work is motivated by the fact that different customers typically place different values on the speed with which their orders are filled, a prevalent theme in the time-based competition literature (see Stalk, 1988; Blackburn, 1991). Recognizing such customer differences naturally leads to the consideration of differential pricing and operations strategies based on delivery speed. Following this line of reasoning, in this paper we consider a situation in which the price of a good offered to consumers can vary in correspondence with different delivery lead time options, including the option of immediate demand satisfaction in make-to-stock systems. We develop a model that captures the operations cost benefits and profit implications that result from such pricing strategies. Our modeling approach determines the best prices to offer for “make-to-stock” demands (which request immediate demand satisfaction), as well as demands that allow positive lead-times.

In the most fundamental form of our model, a supplier provides two pricing options: a “make-to-stock” (MTS) option, where in-stock items are offered at some base price, and a discounted “make-to-order” (MTO) option, where the item is delivered to the customer following procurement from an external supplier. Although such a strategy may potentially reduce the supplier’s total revenue, as we later show, this may be more than offset by a decrease in inventory-related costs, leading to higher profit. In certain contexts this can be particularly advantageous to suppliers. For example, retailers who primarily sell through on-line channels (e.g., computer or cellular telephone manufacturers) may also have retail displays that allow consumers to test the physical product. Different customers may have different expectations from such stores: while certain customers may desire immediate demand satisfaction from stock at the store, others who simply wish to try out the product in the store may be willing to accept delayed product delivery in exchange for a reduced price. The model we present can serve to determine appropriate pricing in such contexts, while the associated pricing strategy can lead to increased profit for the supplier when compared with a single-pricing strategy.

The marketing and economics literature has long recognized the value of segmenting markets based on particular customer preferences for product attributes (see, for example, Frank et al., 1972). Growing research interest exists in studying the effects of offering consumers an ever greater number of product attribute options in order to properly segment and target a greater proportion of the entire customer pool. Most work in this line of research focuses on physical attributes such as size and color (see, for example, Mussa and Rosen, 1978; Moorthy, 1984; De Groote, 1994), whereas we focus on price and delivery delay.
A recent stream of literature considers differentiated customer reactions to price and lead time in capacitated manufacturing systems (e.g., Palaka et al., 1998; and So and Song, 1998). These papers illustrate the interplay between capacity utilization, congestion, and the firm’s ability to meet quoted lead times when lower prices increase demand, although they do not target different market segments using different prices. Additional notable work in this vein that considers heterogeneous customer reactions to delivery delays and pricing in queuing systems includes Lederer and Li (1997) and Van Mieghem (2000). In this stream of research, the firm observes continuously arriving demands from different customer classes, and must determine scheduling rules that affect customer delay costs. Pricing decisions affect the rate of demand seen by the queuing system, which in turn, along with production and scheduling policy decisions, influences delays (and delay costs) experienced by customers. In contrast to the model we present, this queuing based research considers only make-to-order contexts, and does not, therefore, consider the inventory cost implications associated with make-to-stock systems.

Chen (2001) studies a context that is closely related to ours, where a monopolist sells a good to a market in which customers may be enticed by a reduced unit price to accept a delay in delivery. The market consists of a finite number of segments with different degrees of aversion to delays. The firm offers a pre-determined price schedule under which each customer self-selects the price they pay with the corresponding delivery time. The article explores the benefits of such a pricing strategy in multistage supply chains by recognizing the advance demand information that can be exploited through customer selection, and develops heuristics for setting a menu of prices. Unlike Chen (2001), we determine the optimal price associated with each delivery plan in a single-stage system, while accounting for the possibility of different shortage costs for each customer class. Our analysis, therefore, allows us to characterize the important revenue and cost drivers that determine optimal price values and the conditions under which such pricing strategies can provide the greatest benefit to a supplier.

Frank et al. (2003) recently analyzed an inventory control problem with two demand classes, one of which is deterministic and the other stochastic. Deterministic demand arises as a result of longer term “contracted” customers whose demand must be met, while a stochastic stream of demands also exists, but whose demand may be lost in the case of a supply shortage. Their model addresses contexts with fixed order costs, and must decide the amount to order and how much stochastic demand to satisfy in each period in order to minimize expected inventory-related costs. The model we discuss is closely related to theirs, except that we focus on optimal pricing decisions when demand is a function of price, and we do not study the impacts of fixed order costs on the optimal inventory policy. Moreover, as we later discuss, the deterministic component of demand we consider arises directly as a result of offering lower prices in exchange for a delay in order satisfaction. We can thus view our approach as a mechanism for converting a segment of the demand pool to longer term contracted demands. In exchange for becoming a long term contracted customer who provides a sufficiently early demand signal, a price reduction is offered and demand satisfaction is guaranteed.

The remainder of this paper is organized as follows. Section 2 presents the basic model in which a stochastic, additive (or linear) component of supplier demand exists. We begin with a situation where the supplier offers a discounted-price, longer-delivery-time option in addition to its basic make-to-stock option. In Section 3, we derive optimal pricing decisions under a stable market responses to price, assuming the basic additive demand model presented in Section 2. Section 4 then considers the model implications under an alternative multiplicative stochastic demand model. Section 5 contains a numerical analysis of our policy results, and Section 6 discusses concluding remarks and directions for potential future research.

2. Problem description and modeling approach

Consider a supplier that sells a good directly to consumers on a recurring basis over an effectively infinite horizon (we use the term supplier to refer to any stage that satisfies customer demands on a recurring basis).
The supplier currently uses one mode to satisfy consumer demands, which consists of stocking the good on the shelf in every period and offering a single price to all consumers. That is, when a demand occurs and the item is in stock, the customer receives the product immediately at a unit base price of $p_b^0$. When the item is not available in stock, the demand is backordered and when it is ultimately satisfied, the consumer still pays the base price of $p_b^0$, although the supplier incurs a penalty cost for each backordered unit. Letting $c$ denote the supplier’s unit procurement cost, we define $p_b = p_b^0 - c$ as the net base price paid by the customer. In our subsequent analysis it will be convenient for us to translate any price $\hat{p}$ to its equivalent net price $p = \hat{p} - c$.

Under the single-base-price approach, we first consider the situation in which customer demand in period $t$ is a downward-sloped function of base price plus a random factor.\(^\text{1}\) We use $O_t$ to denote demand in period $t$ and let $O_t = \mu(p_0) + \epsilon_t$, where $\mu(p_0)$ is some guaranteed minimum demand level, which is a deterministic demand component solely dependent upon the net base price.\(^\text{2}\) We assume that $\mu(\cdot)$ is monotone decreasing and concave, i.e., $\mu'(\cdot) \leq 0$ and $\mu''(\cdot) < 0$ with $\mu'(0) = 0$, and that a reservation price $\bar{p}$ exists at which $\mu(\bar{p}) = 0$ and $\mu'(\bar{p}) = -\infty$. The random demand factors in successive periods are iid, following a distribution with pdf $f(\cdot)$ on support $[0, B]$ for some $B > 0$. We assume demand per period has mean $\mu$ and variance $\sigma^2$, and use $\epsilon_t, L$ (with density $f_{\epsilon_t, L}(\cdot)$) to denote the Lth convolution of $\epsilon_t$ with itself, representing random lead-time demand from period $t - L$ through period $t - 1$.

Let $L_o \geq 0$ denote the supplier’s external replenishment lead time. In order to derive insights and for analytical tractability, we assume that the supplier’s procurement source has unlimited capacity. Clark and Scarf (1960) show that under these assumptions an order-up-to $y$ inventory policy is optimal. That is, if the inventory position at the beginning of a period is lower than $y$, we order enough to bring the inventory position up to $y$. We next characterize the supplier’s expected profit when it implements such an inventory policy and chooses the best base price to attract customers. We later use the results from this base-case analysis to evaluate the benefits of an additional lead-time and price option.

2.1. Base-case analysis

We first derive the long-run average profit per period under the operating mode in which the supplier uses a strict MTS\(^\text{3}\) approach with a single price. Let $D_t$ denote the demand in period $t$ and $L_d$ the planned delivery lead time, which is the time interval between an order placement and final demand satisfaction. Under the current operating mode, $L_d = 0$ and all customer orders translate into demands in the same period, i.e., $D_t = O_t$. Shortages are backlogged at a cost of $b$ per unit (as we later discuss, this shortage cost may represent an average shortage cost among all customers, with each customer possibly having a unique shortage cost). The supplier also incurs a holding cost $h$ for each unit remaining at the end of the period. In response to the two demand components, the order-up-to level $y$ can be rewritten as $y = L_o \mu(p_0) + y_0$, where $y_0$ is the target stock level for the random lead-time demand component, $\epsilon_{t, L}$.

The supplier’s expected cost per period consists of two parts, variable costs and inventory-related costs. Since the supplier implements an order-up-to policy, the replenishment order in a period equals the customer demand in the previous period. The expected purchasing cost per period is thus constant under a stationary demand process. On the other hand, additional variable costs may be incurred for each unit actually sold. Since the price is the net unit revenue over and above such variable costs, and all shortages are

\(^{1}\) In addition to this additive demand model, Section 4 considers a multiplicative demand model.

\(^{2}\) Since there is a one-to-one correspondence between net price and the true price (through a linear translation), we assume that either the supplier characterizes its demand as a function of net price, or that the function $\mu(p_0)$ maps the price $p_b$ to its equivalent true price before computing the resulting demand level. That is, if $\mu(p_b^0)$ denotes the demand as a function of the true price, then $\mu(p_b^0) = \mu(p_b^0 + c) = \mu(p_b^0)$. In the remainder we take the generic term price to imply the net price, and the function $\mu(\cdot)$ to imply the demand at the associated true price.

\(^{3}\) Although we use the term “make-to-stock”, this idea applies to “order-to-stock” contexts frequently found in retail sales contexts.
backlogged, we no longer explicitly model the supplier’s variable costs. To quantify inventory-related costs, we denote $D_{t,L_s}$ as lead-time demand from period $t = L_s$ through period $t - 1$, where $D_{t,L_s} = L_s\mu(p_0) + \varepsilon_t L_s$.

The expected inventory cost per period equals $hE(y - D_{t,L_s})^+ + bE(y - D_{t,L_s})^-$, which is equal to $hE(y_0 - \varepsilon_t L_s)^+ + bE(y_0 - \varepsilon_t L_s)^-$ by substitution. The intuition behind this transformation is that the supplier can always satisfy the deterministic portion of demand $L_s$: $\mu(p_0)$ and avoid any inventory costs for this demand component. Letting $\Pi_0(p_0, y_0)$ denote the expected profit per period, we have

$$\Pi_0(p_0, y_0) = p_0\mu(p_0) + p_0\bar{\mu} - bL_s\bar{\mu} + by_0 - (h + b)A(y_0),$$

where $A(y) = \int_0^y (y - x)f_{L_s}(x)dx$ is the excess function. The following result characterizes the optimal solution to this profit function.

**Property 1.** $\Pi_0(p_0, y_0)$ is jointly concave in $(p_0, y_0)$. The optimal solution $(p_0^*, y_0^*(b, h))$ satisfies

$$F_{L_s}(y_0^*(b, h)) = \frac{b}{b + h} \text{ and } \mu(p_0^*) + p_0^*\mu'(p_0^*) + \bar{\mu} = 0.$$

The proof of this result is straightforward and hence omitted. We call $p_0^*$ the optimal base price, which, as the stationary condition indicates, does not depend on any parameters other than expected lead-time demand. The optimal stationary target inventory level is $y_0^*(b, h) = L_s\mu(p_0) + y_0^*(b, h)$; thus, the optimal base profit depends on $b$ and $h$ in addition to price. To show this dependency, at the optimal order-up-to level $y_0^*(b, h)$, we write $\Pi_0(p_0^*, b, h) = R(p_0^*) - C_0(b, h)$, where $R(p_0^*) = p_0^*\mu(p_0) + p_0^*\bar{\mu}$ and $C_0(b, h) = bL_s\bar{\mu} + (h + b)A(y_0^*(b, h)) - by_0^*(b, h)$. It is straightforward to show that in the base system, for nonnegative unit holding and inventory costs, $y_0^*(b, h)$ is monotone increasing in $b$ and decreasing in $h; C_0(b, h)$ is monotone increasing in both $b$ and $h; \Pi_0(p_0^*, b, h)$ is monotone decreasing in $b$ and $h; C_0(b, h)$ is concave in $b$ and $h$; and $\Pi_0(p_0^*, b, h)$ is convex in $b$ and $h$ for fixed $p_0^*$.

Recall that the quantity $R$ is the net expected revenue per period. If there is no uncertainty in demand, $y_0^*$ can be set to be exactly the amount of lead-time demand $L_s\bar{\mu}$. In this case, no inventory-related cost is incurred and $R$ is also the (expected) net profit. Demand uncertainty, of course, reduces expected profit and $R$ thus provides an upper bound on the expected profit per period. The expected net revenue $R$ is not sensitive to changes in cost parameters, while $C_0(b, h)$ is affected by both $b$ and $h$. Since the supplier will not operate otherwise, we henceforth assume that $\Pi_0(p_0^*, b, h) \geq 0$ in all problem instances we consider.

### 2.2. Impact of offering a lower-price-delayed-demand option

Suppose that in addition to satisfying demands at the base price $p_0^1$ (the superscript 1 is used to distinguish from $p_0$ in the base case) with zero delivery time, the supplier announces a second price-delivery option in which it guarantees, in exchange for a reduced price, delivery of customer orders after $L_s$ periods but at a lower price $p_0^1 < p_0^0$. We refer to this as a lower-price-delayed-demand (LPDD) option. We assume that the market for the supplier’s product remains relatively stable. In other words, there is a negligible change in market size after introducing the LPDD option. (We later discuss the implications of relaxing this assumption in Section 3.) Given two alternatives instead of one, customers can now either stick with the base-price-zero-lead-time option or choose the LPDD option. We assume that the fraction of those customers who stick with the base option is $z_0$, which is influenced by the relative magnitude between $p_0^1$ and $p_0^0$. We refer to $z_0$ as the market split coefficient.\(^4\)

\(^4\) As with the expected demand function, we assume that $z_0$ is a function of the net price ratio, which we define as $x_1 = p_0^1/p_0^0$. Alternatively, we might assume that we can use a contraction mapping to map any net price ratio to its equivalent true price ratio, and compute the associated market split function value. This latter approach does not affect the structural results from our subsequent analysis.
2.2.1. Periodic demand

Under the LPDD option, customers fall into two classes. MTS (make-to-stock) customers are those who choose the original option, while MTO (make-to-order) customers are those who accept the LPDD option. Orders from MTO customers in any period form a portion of the demand seen (and to be satisfied) later. Since this class of demands provides the supplier with sufficient warning, the supplier can ensure satisfying them without any shortages (and without the need to hold inventory in advance) by scheduling the replenishment of these orders to arrive exactly when they are needed. Uncertainty remains for demands from MTS customers. We denote orders from the two classes in period \( t \) by \( O_{t,1} \) and \( O_{t,2} \) respectively, where \( O_{t,1} = x_0 \cdot O_t \) and \( O_{t,2} = (1-x_0) \cdot O_t \). Demand in period \( t \) can be written as

\[
D_t = O_{t,1} + O_{t-L_d,2} = x_0 \cdot O_t + (1-x_0) \cdot O_{t-L_d} = x_0 \mu + (1-x_0)(\mu + \hat{e}_{t-L_d}) + x_0 e_t = \mu_e + x_0 \cdot e_t,
\]

where \( \mu_e = x_0 \mu + (1-x_0)(\mu + \hat{e}_{t-L_d}) \) is the deterministic portion of demand in the current period. We use \( \hat{e}_{t-L_d} \) to denote the realization of the random error term \( e_{t-L_d} \) to emphasize that this quantity is known at the beginning of period \( t \).

2.2.2. Replenishment process

To satisfy demands from different classes, the supplier strictly makes to stock for MTS customers and makes to order for MTO customers. We further assume that when MTS customer shortages occur, they are strictly backlogged and satisfied upon the supplier’s receipt of the next shipment from its source. Observe that with the LPDD option, the backlogging cost is only assessed against MTS demands. MTO customers inherently expect a fixed waiting time for a lower price with guaranteed full satisfaction following the delay; thus, shortage costs do not apply to these orders. Taking into account the supplier’s lead time of \( L_s \), at the beginning of period \( t \), the supplier needs to order exactly \( O_{t-(L_d+L_s),2} \) for MTO orders placed in period \( t-L_d + L_s \) and to be satisfied in period \( t+L_s \). At the same time, the supplier’s order must also buffer against uncertainty associated with future MTS demands.

The class of inventory management policies we consider assumes that stock ordered for MTO customers is either shipped directly to these customers (and thus never physically handled by the supplier), or is held as reserved stock for these customers and cannot be used to satisfy MTS demands. In certain contexts, it may be optimal to pool this inventory, particularly since MTS customers may have higher shortage costs. Our model, however, does not allow this, and therefore may apply to contexts in which items are either directly shipped to consumers, or the supplier has committed to offering a policy that guarantees delivery to those customers who are willing to delay demand satisfaction. That is, in addition to a reduced price, customers have the additional incentive of guaranteed demand satisfaction following the prescribed delay. As we noted earlier, this may viewed alternatively as a mechanism to establish a deterministic pool of “contracted” demands whose revenues are guaranteed to the supplier in every period. Since these demands bring a guaranteed source of revenue in every period without associated inventory costs, the supplier agrees not to use the corresponding stock to satisfy MTS customers.

With respect to shortage costs, Chen (2001) observed that the relative value of shortage costs for different customer classes is not always clear. That is, while MTS customers are clearly more time sensitive, the loss of goodwill for a MTO customer in response to a shortage following a guaranteed delay period may actually be equal to or higher than for MTS customer shortages. Chen (2001) thus assumed equal shortage costs for all customer classes. In contrast, we consider the model implications when MTS shortage costs are at least as high as those for MTO, since MTS customers have greater time sensitivity.

2.2.3. Profit analysis

As mentioned previously, the supplier may incur a variable cost for each item sold. We assume that the variable costs incurred are the same for all items regardless of the customer class for which they are destined. This implies that every price represents the net unit profit after variable cost. Given any realization
of \( \hat{\epsilon}_{t-L_t} \), the MTO stock in period \( t \) brings a total revenue of \( p^1_t \cdot (1 - x_0) (\mu(p^0_t) + \hat{\epsilon}_{t-L_t}) \) without incurring any inventory costs. The expected profit from this stock is therefore \( p^1_t \cdot (1 - x_0) (\mu(p^0_t) + \bar{\mu}) \). We next analyze the inventory cost and expected profit associated with the MTS or buffer stockpile. Taking into account supplier lead time, we let \( X^z_{t-L_t} \) represent total random lead time demand from period \( t - L_t \) through period \( t - 1 \), i.e., \( X^z_{t-L_t} = x_0 \sum_{k=t-L_t}^{t-1} \hat{\epsilon}_k \) and use \( f^z_{L_t}(\cdot) \) and \( F^z_{L_t}(\cdot) \) to denote its pdf and cdf. The excess function for \( X^z_{t-L_t} \) can be written as \( z_0 \cdot A(\frac{x}{z_0}) \) for a target inventory level of \( y \). To see this, note that

\[
\int_0^y (y-x)f^z_{L_t}(x)\,dx = \int_0^y (y-x) \frac{z_0}{z_0} f_{L_t} \left( \frac{x}{z_0} \right) \,dx = z_0 \int_0^y \frac{x}{z_0} - z \right) f_{L_t}(z) \,dz = z_0 A \left( \frac{y}{z_0} \right).
\]

Let \( b_1 \) and \( b_2 \) denote the respective penalty costs for each unit of backlogged demand from MTS and MTO customers, with \( b_1 \geq b_2 \). Although in our model customers who choose the MTO option do not experience stockouts, we define \( b_2 \) as the shortage cost associated with these customers’ demands in the case that no MTO option is available, i.e., in the base case where only the MTS option is available. In the base case, the appropriate value of \( b \) should be a weighted average of \( b_1 \) and \( b_2 \), depending on the proportions of MTS and MTO customers. We later approximate the single value of \( b \) to use in the base case as \( b = x_0 b_1 + (1 - x_0) b_2 \). That is, a higher value of \( x_0 \) at optimality implies greater time sensitivity in the market, and a higher shortage cost should be assessed, while a lower \( x_0 \) implies less relative time sensitivity in the market. By appropriately setting \( b_1 \) and \( b_2 \) we thus approximate the distribution of time sensitivity of customers in the market.

Denote \( y^1 \) as the inventory order-up-to level for the buffer stockpile. As with the base case, let \( y^1 = x_0 L_s \mu(p^0_t) + y^0 \). The expected inventory cost per period for the buffer stockpile is \( E[h(y_0^1 - X_{t-L_t})^+ + b_1 (y_0^1 - X_{t-L_t})^-] \). Under our backlogging assumption, the expected sales from MTS customers per period equal average demand. Hence the expected profit from MTS customers is

\[
x_0 p^1_t \mu(p^0_t) + \bar{\mu} + b_1 y^0_t - x_0 \left[ b_1 L_s \bar{\mu} + (h + b_1) A \left( \frac{y^1_0}{x_0} \right) \right].
\]

The expected total profit per period, denoted by \( \Pi(p^0_t, p^1_t, y^0_t, b_1, h) \), is thus,

\[
\Pi(p^0_t, p^1_t, y^0_t, b_1, h) = R_1(p^0_t, p^1_t) + R_2(p^0_t, p^1_t) - C^1(p^0_t, p^1_t, y^0_t, b_1, h),
\]

where

\[
R_1(p^0_t, p^1_t) = x_0 [\mu(p^0_t) + \bar{\mu}],
\]

\[
R_2(p^0_t, p^1_t) = p^1_t (1 - x_0) [\mu(p^0_t) + \bar{\mu}],
\]

and

\[
C^1(p^0_t, p^1_t, y^0_t, b_1, h) = b_1 L_s \bar{\mu} x_0 - b_1 y^1_0 + (h + b_1) x_0 A \left( \frac{y^1_0}{x_0} \right).
\]

The terms \( R_1 \) and \( R_2 \) are the expected revenues from MTS and MTO customers, respectively, while \( C^1 \) is the expected inventory cost incurred. We first characterize the optimal inventory decision for given values of \( p^0_t \) and \( p^1_t \), and the associated \( x_0 \). Let \( y^1_* \) denote the order-up-to level that maximizes \( \Pi_1(p^0_t, p^1_t, y^0_t, b_1, h) \). We thus have the following property.

**Property 2.** Given prices \( p^0_t \) and \( p^1_t \) with the associated \( x_0 \),

1. \( \Pi_1(p^0_t, p^1_t, y^0_t, b_1, h) \) is concave in \( y^1_* \) for fixed prices and \( b_1 \) and \( h \);
2. \( y^1_0(b_1, h) = x_0 y^0_0(b_1, h) \), and \( y^1_0(b_1, h) = x_0 L_s \mu(p^0_t) + y^0_0(b_1, h) \);
3. \( C^1(p^0_t, p^1_t, y^0_t, b_1, h) = x_0 C_0(b_1, h) \);
4. \( \Pi_1(p^0_t, p^1_t, y^0_t, b_1, h) = x_0 \Pi_0(p^0_t, b_1, h) + (1 - x_0) p^1_t [\mu(p^0_t) + \bar{\mu}] \).
Part 4 of Property 2 shows that with the introduction of the MTO alternative, given the prices for each alternative, the expected profit per period is essentially a convex combination of the expected base profit (offering a single-price-zero-waiting-time option with the same penalty cost and holding cost parameters) with all MTS customers, and the riskless profit (net revenue) from MTO customers; the weight is determined by the magnitude of the market split coefficient. Assuming that the difference in shortage costs between MTS and MTO customers is not too great, parts 2 and 3 of the property illustrate the potential inventory and cost reduction with respect to the base case (the actual inventory and cost reduction depend on the appropriate choice of the shortage cost $b$ in the base case, as we later discuss). The potential benefit of the LPDD option therefore depends on the relative shortage costs and the prices offered, and how these in turn affect the market split coefficient. As we will show later, under certain assumptions on the structure of the market split coefficient, the optimal strategy can substantially increase expected profit with respect to that in the current strict MTS mode.

### 3. Optimal decisions under additive demand model

This section characterizes system behavior under the LPDD option in a stable market. We assume that the planned delivery lead time for MTO demands is set exogenously, which can be imposed based on either the actual production and transportation time or explicit knowledge of customer tolerances. The supplier optimizes its expected profit per period by choosing the best prices for the two classes of customers. It is intuitive that the higher the price that the supplier charges for the LPDD option, the higher the proportion of MTS customers (and hence the lower the value of $z_0$).

In addition to the market split coefficient $z_0$; we define the diversion proportion $z_1$, as the fraction of MTO customers. Under our assumption of a stable market size, $z_1 = 1 - z_0$. We assume that $z_0$ and $z_1$ are both functions of the price ratio, and we define for convenience $x_1 = \frac{p_1}{p_0}$ as the discount ratio for the second option. To simplify the presentation of our expected profit function and subsequent results, we define the profit ratio of the system under a single-price-zero-waiting option (for given penalty and holding costs), denoted by $k(p_0, b, h)$, as follows:

$$k(p_0, b, h) = \frac{\Pi_0(p_0, b, h)}{\Pi_0(p_0, b, h) + C_0(b, h)}.$$  

Observe that the profit ratio $k(p_0, b, h)$ is between 0 and 1 and is decreasing in $b$. By Property 2, the expected profit under the LPDD option at the optimal order-up-to level $y_0$ can be written as a function of $p_0^i$ and $x_1$ (where we replace the $p_0^i$ argument by $x_1$ and suppress the $y_0^i$ argument in $\Pi_1$) as

$$\Pi_1(p_0^i, x_1, b_1, h) = \Pi_0(p_0^i, b_1, h) + R(p_0^i)[(x_1 - k(p_0^i, b_1, h))z_1(x_1)].$$

Since $z_0 + z_1 = 1$, the value of either one of these implies the other. We study $z_1$ as a function of $x_1$ and make the following general structural assumptions, which we would expect to see in practice, and assume hold throughout the remainder of this section:

A1. $z_1$ is strictly decreasing and concave in $x_1$, i.e., $\frac{\partial^2 z_1}{\partial x_1^2} < 0$ and $\frac{\partial z_1}{\partial x_1} < 0$.

A2. $z_1(1) = 0$, $z_1(0) = 1$, and $0 < z_1(x_1) < 1$ for $0 < x_1 < 1$.

Assumption A1 states that with an increase in the discount ratio, fewer customers can be attracted to the second option, and the marginal rate of such change is diminishing. We assume concavity of $z_1$ for analytical tractability and because we might expect that a discount would initially induce a large number of customers to defect to the LPDD option, while the number of those defecting might taper off as the discount becomes very large (e.g., nearly all “defectors” may have defected at a 50% discount, while greater
discounts will induce fewer and fewer customers). A2 stipulates boundary conditions that imply all customers can be induced to wait for delivery when \( x_1 = 0 \), while at the other extreme, all customers stick to the original option if no price incentive is given to customers who may be willing to wait. Next we analyze the properties of optimal solutions under this market split function. The following lemma is necessary to later characterize the resulting optimal prices.

**Lemma 1.** Under the additive demand model, under assumptions A1 and A2, and for any \( p_0^1 \), there exists a unique optimal discount ratio \( x_1(p_0^1) \) such that \( k(p_0^1, b_1, h) < x_1(p_0^1) < 1 \).

Lemma 1 illustrates that the less profitable the supplier is in the base case, the greater the incentive to offer the discounted LPDD option. To determine the optimal prices, observe that given \( p_0^1 \), the optimal discount ratio satisfies the following first-order condition.

\[
\alpha_1(x_1(p_0^1)) + (x_1(p_0^1) - k(p_0^1, b_1, h)) \frac{\partial \alpha_1}{\partial x_1(p_0^1)} = 0.
\]

Substituting the optimal discount ratio \( x_1^* \) into the profit function, we can write \( \Pi_1 \) as a function of \( p_0^1, b_1, \) and \( h \) as

\[
\Pi_1(p_0^1, b_1, h) = p_0^1[\mu(p_0^1 + \bar{\mu})][1 + (x_1(p_0^1) - k(p_0^1, b_1, h))\alpha_1(x_1(p_0^1))] - C_0(b_1, h).
\]

By the Envelope Theorem, we can show that

\[
\frac{\partial \Pi_1}{\partial p_0^1} = [\mu(p_0^1 + \bar{\mu} + p_0^1\mu'(p_0^1))] \cdot [1 + (x_1(p_0^1) - 2k(p_0^1, b_1, h))\alpha_1(x_1(p_0^1))].
\]

The following result characterizes the optimal solution to the supplier’s problem when offering two options to customers in a stable market.

**Property 3.** Under a stable market reaction with additive demand, \( \Pi_1(p_0^1, b_1, h) \) is concave in \( p_0^1 \) and the optimal decision \((p_0^1, p_1^*, y_0^*)\) satisfies the following:

1. \( p_0^1 = p_0^* \).
2. \( p_1^* = x_1^* \cdot p_0^* \) where \( 1 > x_1^* > k(p_0^1, b_1, h) \) and satisfies \( \alpha_1(x_1^*) + (x_1^* - k(p_0^1, b_1, h)) \frac{\partial \alpha_1}{\partial x_1^*} = 0 \).
3. \( y_0^*(b_1, h) = (1 - \alpha_1(x_1^*))y_0^*(b_1, h) \).

Property 3 indicates that under a stable market reaction, we extract the same unit revenue from MTS customers as in the base case; this revenue is supplemented, when profitable, with the riskless revenue from MTO customers. Surprisingly, even though we separate the market based on time sensitivity, the price charged to the more time sensitive MTS customers does not change when offering the LPDD option (in the additive model, the optimal price is not a function of the shortage cost; as we will later see, this is not the case under a multiplicative demand model). The optimal discount ratio, \( x_1^* \), is bounded from below by the base case profit ratio when the shortage cost equals \( b_1 \), the shortage cost of the time sensitive customers. In particular, using part 3 of Property 3, we can write the optimal discount ratio as

\[
x_1^* = k(p_0^1, b_1, h) - \frac{\alpha_1(x_1^*)}{\frac{\partial \alpha_1}{\partial x_1^*}}.
\]

Since the profit ratio \( k(p_0^1, b_1, h) \) is decreasing in \( b_1 \) (and \( \alpha_1(*) \) is concave and decreasing in \( x_1 \)), we see that, all else being equal, the greater the time sensitivity of the MTS customers (as measured by an increasing value of \( b_1 \)), the lower the discount price offered to the MTO customers. Thus a high degree of time sensitivity in one segment of the market can actually induce the supplier to offer lower prices to the segment
with time delivery flexibility (this lower price is required to induce a greater percentage of the market to accept a delivery delay; those customers who were willing to wait in the first place receive additional benefits as a result of other customers’ preferences).

For completeness, note that we can write the optimal expected profit as a function of \( b_1 \) and \( h \) (at the optimal value of \( p_0^* \)) as

\[
\Pi_1(b_1, h) = (R - C_0(b_1, h)) \cdot (1 - x_1(x_1^*)) + R \cdot x_1^* x_1(x_1^*).
\]

Clearly, offering the LPDD option is justified if and only if the resulting optimal total profit is larger than \( \Pi_0(b, h) \), where the proper penalty cost to use in calculating \( \Pi_0(b, h) \) is the weighted average \( b = (1 - x_1^*)b_1 + x_1^*b_2 \) where \( x_1^* = x_1(x_1^*) \). The profitability of the LPDD option is therefore affected by the relative values of \( b_1 \) and \( b_2 \), as is further illustrated by the following property.

**Property 4.** Under a stable market reaction, for any \( b_1 \), there exists a \( b_2(b_1) \) such that \( 0 < b_2(b_1) < b_1 \) and \( \Pi_1(b_1, h) \leq \Pi_0(b, h) \) for \( 0 < b_2(b_1) < b_2(b_1) \).

Thus, even though the optimal MTS price is not affected by the magnitude of shortage costs, both the MTO price (see Lemma 1) and the relative profitability of the new practice rely heavily on the relative shortage costs of the customer classes.

Due to space limitations, our analysis in this paper relies on a stable market reaction assumption, which effectively assumes that all members of the original market either stick with the initial price offer or defect to the LPDD option. That is, we did not consider contexts in which some customers might actually choose not to purchase the item at all upon learning about the LPDD option (since such consumers might perceive that the brand has cheapened, or that the value of the item has decreased). Nor did we consider the situation where new customers are enticed by the new offer. In Jiang and Geunes (2004) we address a wider range of potential consumer preferences and behavior in practice by utilizing a more general modeling approach with respect to market reaction. In particular, we consider cases where \( z_0 + x_1 > 1 \) (and the market expands with new customers being attracted to the supplier) and where \( z_0 + x_1 < 1 \) (the market contracts). The results show that for the additive demand model, even under these more general market reaction assumptions we still have \( p_0^* = p_0 \) at optimality, i.e., the supplier does not charge a premium to MTS customers, despite their higher shortage costs. While these models with more general market reactions do not allow us to derive closed-form expressions on expected profit, we establish conditions under which the LPDD option will lead to increased expected profits in Jiang and Geunes (2004).

### 4. Optimal decisions under multiplicative demand model

The results in the previous sections are based on a demand model form \( O_t = \mu(p_0) + e_t \). It is thus interesting and important to know if, or to what degree, those results may change under other demand models. To this end, we study the following multiplicative demand model, \( O_t = \mu(p_0) \cdot e_t \), where \( \mu(\cdot) \) is defined as in the additive demand case. The random demand components \( e_t \) are iid, following the distribution \( f(\cdot) \) with support on \([0, B]\), for some \( B > 0 \) with mean \( \bar{\mu} > 0 \). We also use \( e_{t,L} \) with pdf \( f_L(\cdot) \) as its density function to denote the \( L \)th convolution of \( e_t \) with itself. Note that lead-time demand from period \( t - L \) through period \( t - 1 \) under this model can be written as \( D_{t,L} = \mu(p_0)e_{t,L} \). All other problem settings are defined as in the additive demand model case.

In the base case \( D_t = O_t \) and the supplier sets its target inventory position \( y \) to buffer against lead-time demand. Accounting for the two demand components, the order-up-to level can be written as \( y = \mu(p_0) \cdot y_0 \), where \( y_0 \) is called the stocking factor (see Petruzzi and Dada, 1999). Expected sales per period equals \( \mu(p_0)\bar{\mu} \), and the expected inventory cost per period is \( hE(y - D_{t,L})^+ + bE(y - D_{t,L})^- \), which equals \( \mu(p_0)[hE(y_0 - e_{t,L})^+ + bE(y_0 - e_{t,L})^-] \). \( \Pi_0(p_0, y_0, b, h) \), the expected profit per period, can be written as
Given our profit ratio definition, which can be equivalently written as 

$$p_0 \mu(p_0) \tilde{\mu} - \mu(p_0)[bL, \tilde{\mu} - by_0 + (h + b)A(y_0)]$$

It is easy to verify that \( \Pi_0(y_0) \) is jointly concave in \((p_0, y_0)\) and the optimal \( y_0^* \) satisfies \( F_{L_2}(y_0^*) = \frac{1}{y_0^*} \). Note that the optimal target stocking factor in the multiplicative demand model is the same as that in the additive demand model. We also define \( C_0(b, h) = bL, \tilde{\mu} + (h + b)A(y_0^*) - by_0^* \), and note that \( C_0 \) is increasing and concave in \( b \), and the optimal inventory cost given \( p_0 \) is thus \( \mu(p_0)C_0(b, h) \). By substitution, at \( y_0^* \) (and suppressing the dependence of \( \Pi_0 \) on \( y_0 \)), the expected profit function can be written as

$$\Pi_0(p_0, b, h) = \mu(p_0)[p_0\tilde{\mu} - C_0(b, h)].$$

\( \Pi_0(p_0, b, h) \) is concave in \( p_0 \) and the optimal price \( p_0^* \) satisfies

$$\mu(p_0^*)\tilde{\mu} + p_0^*\mu'(p_0^*)\tilde{\mu} - C_0(b, h)\mu'(p_0^*) = 0.$$  \( \text{(1)} \)

In contrast to the additive demand case, the optimal base price in the multiplicative demand model depends on both \( b \) and \( h \). It can be shown that \( \Pi_0 \) has increasing differences in \( p_0 \) and \( b \), so \( p_0^*(b, h) \) increases in \( b \). The optimal stationary order-up-to level is \( y^* = \mu(p_0^*)y_0^* \), and we write the optimal base profit as \( \Pi_0(b, h) = R(b, h) - \mu(p_0^*)C_0(b, h) \), where \( R(b, h) = p_0^*\mu(p_0^*)\tilde{\mu} + C_0(b, h) = bL, \tilde{\mu} + (h + b)A(y_0^*) - by_0^* \). The quantity \( R(b, h) \) is the expected revenue per period.

Following our previous analysis, under the added LPDD option, we can write expected profit per period, denoted by \( \Pi_1(p_0^*, p_1^*, y_0^*, b_1, h) \), as

$$\Pi_1(p_0^*, p_1^*, y_0^*, b_1, h) = R_1(p_0^*, p_1^*) + R_2(p_0^*, p_1^*) - C_1(p_0^*, p_1^*, y_0^*, b_1, h),$$

where \( R_1(p_0^*, p_1^*) = x_00(p_1^*)\mu(p_0^*)\tilde{\mu} \), \( R_2(p_0^*, p_1^*) = p_1^*(1 - x_0)\mu(p_0^*)\tilde{\mu} \), and \( C_1(y_0^*, p_0^*, p_1^*, b_1, h) = \mu(p_0^*)[bL, \tilde{\mu} - x_0(y_0^*) - (h + b)A(y_0^*) - by_0^*] \). \( R_1 \) and \( R_2 \) are the expected revenues from MTS and MTO customers, respectively, while \( C_1 \) is the expected inventory-related cost. Given \( p_0^* \) and \( p_1^* \) with the associated \( x_0 \), the following result parallels Property 2 and characterizes optimal inventory decisions under the multiplicative demand model.

**Property 5.** Under the multiplicative demand model, given \( p_0^* \) and \( p_1^* \) with the associated value of \( x_0 \),

1. \( \Pi_1(p_0^*, p_1^*, y_0^*, b_1, h) \) is concave in \( y_0^* \) for given prices and \( b_1 \) and \( h \).
2. \( y_0^*(b_1, h) = x_0y_0^*(h, b_1) \) and \( y_1^*(b_1, h) = x_0\mu(p_0^*)y_0^*(b_1, h) \).
3. \( \Pi_1(p_0^*, p_1^*, y_0^*, b_1, h) = x_0\Pi_0(p_0^*, b_1, h) + (1 - x_0)p_1^*\mu(p_0^*)\tilde{\mu} \).

To compare the multiplicative demand model with the additive one considered previously, we consider a simple linear split coefficient function that equals the discount ratio, i.e., \( x_0 = \frac{p_1^*}{p_0^*} \). We already know that the optimal order-up-to level for MTS demands is \( x_0y_0^*(b_1, h) \), and the corresponding inventory cost is \( x_0\mu(p_0^*)C_0(b_1, h) \). The expected profit function under the simple market split coefficient at \( y_0^* \) is

$$\Pi_1(p_0^*, p_1^*, b_1, h) = \mu(p_0^*) \left[ p_1^*\tilde{\mu} - \frac{p_1^*}{p_0^*}C_0(b_1, h) + p_1^*\left(1 - \frac{p_1^*}{p_0^*}\right)\tilde{\mu} \right].$$

Given \( p_1^* \), using the necessary and sufficient first-order optimality condition, we get an optimal discount price \( p_1^* \) of

$$p_1^*(p_0^*) = p_0^* - \frac{1}{2} \frac{C_0(b_1, h)}{\tilde{\mu}} = p_0^* - \frac{p_1^*}{2} \frac{\mu(p_0^*)C_0(b_1, h)}{R(p_0^*)}.$$  \( \text{(2)} \)

Applying our profit ratio definition, which can be equivalently written as \( k(p_0^*, b_1, h) = \mu(p_0^*)C_0(b_1, h) \), and the discount ratio \( x_1 = \frac{p_1^*}{p_0^*} \), we can rewrite the optimal discount ratio given \( p_0^* \) as

$$x_1(p_0^*) = \frac{1}{2} (1 + k(p_0^*, b_1, h)).$$
Since $\Pi_0 \geq 0$ and $C_0(b_1, h) > 0$ when $b_1 > 0$, the above expression implies that $p^*_0 > p^*_1(p^*_0) \geq \frac{1}{2} p^*_0$ and the inequality sign is strict if demand uncertainty exists. Substituting into the profit function, we can write $\Pi_1$ as a function of $p^*_0$, $b_1$, and $h$ only as follows:

$$
\Pi_1(p^*_0, b_1, h) = \frac{\mu(p^*_1)}{4p^*_0} \left[2p^*_0 - C_0(b_1, h)\right]^2 \geq 0.
$$

We next characterize the optimal price $p^*_{0,1}(b_1, h)$ with Property 6.

**Property 6.** Under the multiplicative demand model, with a stable market reaction and linear market split function, given $b_1 > 0$ and letting $b = x_0 b_1 + (1 - x_0) b_2$,

1. $\Pi_1(p^*_0, b_1, h)$ is concave in $p^*_0$ and $p^*_{1,0}(b_1, h) < p^*_0(b_1, h)$.
2. There exists a $0 < b^*_2 < b_1$ such that $p^*_{0,0}(b_1, h) = p^*_0(b^*_2, h)$, with $p^*_{1,0}(b_1, h) < p^*_0(b_2, h)$ for $b^*_2 < b_2 \leq b_1$ and $p^*_{1,0}(b_1, h) > p^*_0(b_2, h)$ for $0 \leq b_2 < b^*_2$.
3. $p^*_{1,0}(b_1, h) < p^*_0(b, h)$ for $b^*_2 < b_2 < b_1$.
4. For $0 \leq b_2 < b^*_2$, exactly one of following two situations occurs:
   (i) There exists a $b^*_2$ with $0 \leq b^*_2 < b^*_2$ such that $p^*_{1,0}(b_1, h) \geq p^*_0(b, h)$ for $0 \leq b_2 \leq b^*_2$ and $p^*_{1,0}(b_1, h) < p^*_0(b, h)$ for $b_2 > b^*_2$.
   (ii) $p^*_{1,0}(b_1, h) < p^*_0(b, h)$ for all $b_2$.

Property 6 indicates that, unlike the additive demand model, when MTS and MTO customers’ shortage costs are similar in magnitude, the optimal price for MTS customers is actually less than that in the base case. We gain some insight as to why this is the case by considering the form of the expected profit function under the LPDD option as given in Property 5, part 3, where expected profit is expressed as a convex combination of the base-case profit and the riskless revenue from MTO customers. Observe that, unlike in the additive demand model case, the price that maximizes expected profit in the base case is no longer equal to the price that maximizes the riskless revenue portion of the profit. Referring to Property 5, part 3, we can view the optimal MTS price under the LPDD option as a convex combination of the prices that optimize the base case and riskless revenues, respectively.

Referring to Eq. (1), we see that the optimal price in the base case is higher than the price that maximizes the riskless revenue (because at the price that maximizes riskless revenue, the derivative is positive), due to the presence of the inventory-related cost term in the first-order condition. That is, since price now affects both revenues and inventory-related costs, a higher price is set at optimality in order to cover the resulting inventory-related costs. As a convex combination of the prices that maximize the base-case profit and riskless revenue, the optimal price under the LPDD option is less than that in the base case. Property 6 further indicates, however, that depending on the relative difference between MTS and MTO shortage costs, it is possible for the optimal MTS price to be greater than or less than the optimal base-case price. In particular, if the MTS shortage cost is excessively high relative to MTO, the supplier may need to extract a higher premium from MTS customers to account for their high shortage costs.

Our numerical analysis, discussed in the next section, showed that the behavior of the multiplicative and additive models was very similar in terms of trends in relative optimal profit and prices. For completeness, the following result characterizes the optimal MTO price and order-up-to level under the multiplicative demand model with the linear market split coefficient function.

**Corollary 1.** Under the multiplicative demand model, when the market reaction is stable and the market split function is linear, the optimal discount ratio and inventory position are $x^*_1(b_1, h) = \frac{1}{2} k(p^*_0 b_1, h)$ and $y^*_{0,1}(b_1, h) = y^*_0(b_1, h) + \frac{1}{2} k(p^*_0 b_1, h)$.
We again see that when the profit ratio is one in the base case, the optimal LPDD inventory and price values reduce to those in the base case. On the other hand, a profit ratio less than one implies some discount will be applied at optimality. Moreover, as in the additive case, the higher the shortage cost $b_1$, the lower the price offered for the LPDD option.

5. Numerical analysis

This section provides a numerical study of the benefits of offering the LPDD option for a supplier. The goal of this study is to gain some insight on the degree of profit improvement a supplier might observe by offering such a policy as a function of the important model parameters. We are primarily interested in the relative influence certain key parameters have on the attractiveness of the two-option policy, and as a by product, we can also get an idea of the magnitude of the potential increase in expected profit. For both the additive and multiplicative stochastic demand models, we present results based on a broad number of parameter settings.

For all of our test instances, we compute the maximum expected profit under the LPDD option, and compare this to the base case where only the single price MTS option is offered. Due to the complexity of the required computations, we limit our numerical analysis to a relatively simple characterization of both the stochastic and deterministic demand properties, which allows us to compute the optimal order up to levels and prices efficiently. That is, we use a deterministic demand component that is linear in the base price $p_0$, determined by the relationship $\mu(p_0) = a - \beta p_0$ for nonnegative scalars $a$ and $\beta$. We consider a one-period external supply lead time ($L_s = 1$) and that the stochastic part of demand is uniformly distributed on the interval $[l, u]$, where $0 \leq l \leq u$. To characterize the market split coefficient function $z_0$ we assume a stable market reaction, and again use the linear form $z_0 = p_1/p_0$, and we use MTS and MTO shortage costs $b_1$ and $b_2$ (with $b_1 \geq b_2$). Our primary metric of interest is the percentage profit improvement over the base-case profit, i.e., $% \text{Profit Improvement} = (\Pi_1(b_1, h) - \Pi_0(b, h))/\Pi_0(b, h)$. In order to provide a fair comparison of the relative profit, for the base case we take the shortage cost to be defined as $b = z_0 b_1 + (1 - z_0) b_2$.

Our primary interest lies in characterizing the $% \text{Profit Improvement}$ metric as certain key parameters change. Those parameters of interest are the holding cost, $h$, the demand uncertainty, which we measure using the distribution range $u - l$, the demand parameters $a$ and $\beta$, and the shortage costs $b_1$ and $b_2$. We therefore chose a base set of values for these parameters, and varied each one individually while holding all others at their base values. The base values we used were $h = 10$, $a = 500$, $\beta = 20$, $u = 1000$ and $l = 0$, $b_1 = 90$ and $b_2 = 40$ (therefore under these base values MTS customers have a critical fractile value of 0.9, while MTO customers have a critical fractile value of 0.8). We varied $h$ over the range between 1 and 15 in increments of 1; $a$ between 500 and 1000 in increments of 50; $\beta$ between 0 and 30 in increments of 5; $b_1$ between 40 and 90 in increments of 10; and $b_2$ between 0 and 90 in increments of 10. To vary the distribution parameters $u$ and $l$, we maintained a mean value of 500, and varied $u - l$ from 0 to 1000 in increments of 100. These settings cover a broad range of experimental values and allow us to observe the $% \text{Profit Improvement}$ behavior over a wide range of values.

Fig. 1 illustrates the results of these experiments under the additive demand model. In addition to reporting the $% \text{Profit Improvement}$ metric for each level of settings, we also show the optimal market split coefficient value which, for our parameter settings, ranged between 0.7 and 1 (i.e., between 70% and 100% of customers stick with the base price zero lead time option). As we would expect, in each case the $% \text{Profit Improvement}$ is inversely related to the market split coefficient value (with the exception of the MTO shortage cost, which we later discuss).

The top two figures illustrate how the holding cost and uncertainty substantially impact the attractiveness of the LPDD option. Since increased values of holding cost and uncertainty drive up inventory-related
costs, the supplier can use the LPDD option to reduce inventory costs, and this mechanism becomes more powerful the higher the percentage of inventory-related costs.

The middle two figures illustrate the fact that higher base-case revenues make the LPDD option less attractive, since a high value of $a$ and a low value of $\beta$ imply a high profit potential in the base case. On the other hand, a low value of the deterministic demand component’s intercept, combined with high demand elasticity make the LPDD option quite attractive. The bottom two figures show the effects of the MTS and MTO shortage costs on the potential profit increase. The figure on the left indicates that as the MTS shortage cost increases, only a small reduction in the delayed demand price (reflected through
$x_0$ is necessary to provide additional profit (less than one-tenth of a percentage reduction in the market split coefficient produces around a 0.7% increase in profit percentage).

It is somewhat counter-intuitive that the relative profit of the LPDD option should improve as the MTS shortage cost $b_1$ increases. Note, however, that an increase in $b_1$ may also increase $b$, the base-case shortage cost (since $b = x_0b_1 + (1 - x_0)b_2)$. If the increase in $b_1$ is great and the resulting market split decrease is small (as shown in the bottom left figure), the net result of the increase in $b_1$ is that the base-case profit decreases by a greater amount than the optimal profit under the LPDD option. As the figure on the bottom right indicates, the MTO shortage cost has no effect on the optimal prices ($x_0 = p_1/p_0$ is flat); however, as $b_2$ increases, the base-case profit decreases, making the LPDD option more attractive. Note that we should interpret the actual percentage profit increases with caution, since these results are highly sensitive to the parameters selected. We also note that our problem settings did not lead to a non-zero threshold value for the MTO shortage cost, below which the base case outperforms the LPDD option, although in extreme cases this is possible (note that the market split function $x_0 = p_1/p_0$ does not satisfy the strict concavity assumption A1; in such cases it is possible that the threshold value for $b_2$ is equal to zero, as is the case in the problem instances we tested).

The results for the multiplicative demand model were quite similar with some minor exceptions. First, the % Profit Improvement metrics were universally and substantially higher in the multiplicative case, primarily due to the use of the same data set but with a multiplicative effect on the demand level. Second, the multiplicative model led to a wider range of market split coefficient values at optimality, with a market split value range between 0.6 and 1.0. Finally, recall that in the multiplicative demand model, the optimal MTS customer price is likely to be less than the optimal base-case price (assuming the difference between $b_1$ and $b_2$ is not extremely high). Because of this we considered the value of the ratio of the optimal base-case price to the optimal MTS customer price under the LPDD option, i.e., $p_0/p_0^{\ast}$, which we expect to be at least one. We found this ratio to be increasing with each parameter that makes the base case less profitable. That is, as the holding cost, distribution uncertainty, and shortage costs increase, we found the base-case to MTS price to be increasing. As the deterministic demand component intercept, $a$, increased, this ratio decreased, and as the slope $\beta$ increased, the ratio also increased. Fig. 2 illustrates the behavior of this ratio in the two select parameters $h$ and $\beta$.

In addition to the price ratio $p_0/p_0^{\ast}$, we also provide the ratio of the MTS to MTO price under the LPDD option (which is also the inverse of the market split coefficient in this special case). As the figure illustrates, we see a price reduction not only for those customers who are willing to delay demand satisfaction, but also for the MTS customers. The flexibility of MTO customers under the multiplicative demand model therefore benefits not only these customers, but MTS customers as well, through reduced prices under the LPDD option. The resulting price discounts increase as a function of the inventory-related costs and

Fig. 2. Optimal price ratios as a function of holding cost, $h$, and deterministic demand component elasticity, $\beta$. 

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decrease in deterministic demand component value. In other words, as the base-case profitability decreases, the price discounts seen by the consumers under the LPDD option increase. When compared with the base case, under the multiplicative demand model, the LPDD option has the potential to benefit not only the supplier, but all consumers (MTS and MTO) as well.

Although we could not analytically show the existence of a non-zero threshold value for the MTO shortage cost $b_2$, below which the base-case outperforms the LPDD option in the multiplicative demand model, we next provide test instances that empirically demonstrate the existence of such a threshold. Our analysis showed that such a threshold value is more likely to exist when MTS shortage costs are roughly of the same magnitude as the holding cost (as our analytical results and experiments thus far have shown, the lower the value of inventory-related costs, the less likely the LPDD option will be to benefit the supplier; we therefore looked for test instances in which the LPDD option was only marginally beneficial). Because of this, we changed the value of $b_1$ from its base level of 90 to 10. We then decreased the MTO shortage cost until the LPDD option and the base case were equally profitable. Fig. 3 plots the ratio of the threshold value of $b_2$ to $b_1$ at our base level parameter settings (with the exception of $b_1 = 10$ and for various values of $a$) for various values of the deterministic demand intercept, $a$. For values of $b_2$ below the dashed line, the base-case scenario outperforms the LPDD option. As the figure shows, even under a relatively low MTS shortage cost, the threshold value of $b_2$ is very low (<10% of $b_1$), which, along with our prior results, indicates that the LPDD option can provide profit improvements over a broad range of parameter values. We note here that these problem instances settings will be unlikely to occur in practice, and were chosen to verify the existence of non-zero threshold values for $b_2$.

6. Discussion and concluding remarks

Our analysis throughout has assumed that one additional LPDD option is made to customers. It may be the case that different customers have differing tolerances for delays. We might therefore suppose that the supplier can offer $n$ LPDD options, each with a different delivery lead-time, in addition to the MTS option. Under the additive demand model and a generalized version of assumptions A1 and A2 for the market split function, and assuming a stable market reaction and exogenously determined delay times, we can show that the optimal base price is still $p_0$, and the optimal discount ratios are strictly decreasing in delay times, but bounded from below by the profit ratio $k(p_0, b_1, h)$ under the base case. In addition, the expected profit of the supplier is increasing in the number of options and bounded from above by $R$, the riskless revenue per
period at \( p^*_0 \). This result is based on our assumption that any new option will attract some positive fraction of MTS customers to the new option. Ultimately, however, this assumes that as the number of low price options increases to infinity, nobody will demand immediate satisfaction and all customers will provide “no cost” demands to the supplier (in terms of inventory-related costs). We believe that these assumptions provide a good approximation to what we might observe in practice under a small number of options with reasonable time delays. At some point, however, the necessary time delays required in order to provide additional options would result in no additional customers selecting such options. Moreover, the cost of coordinating a large number of options has not been accounted for, and thus a broad set of options may not be profitable when accounting for such costs. This analysis does, however, provide interesting insight into the additional value a broad set of pricing options may provide to suppliers.

The fact that consumers have different reservation prices and tolerances for delivery lead time has a variety of implications for suppliers and provides greater opportunities to better match supply with demand and increase profit. In this paper, we studied a practical set of scenarios in which a supplier offers a menu of price-delivery options to consumers. To take advantage of the advance demand information from those consumers who choose to wait for demand satisfaction at a discount price, the supplier adjusts its inventory policies to accommodate two separate stockpiles, one deterministic and the other stochastic, for different demand streams. By using a simple base stock policy for the buffer stockpile and having the deterministic stock level arrive just-in-time to exactly match advance orders, we characterize system performance under a variety of market assumptions and give sufficient conditions under which the new practice will make the supplier better off.

There are a number of possible avenues for extending the work in this paper. The most direct one is to incorporate delivery lead time into consideration. In our current model, consumer delivery lead time is assumed to be determined exogenously and consumers make choices based on prices associated with these pre-determined lead times. In practice, however, the criteria by which consumers trade price for delivery time can be complicated. A model to appropriately reflect the implications of delivery lead time as a decision variable while accounting for its interaction with price is required. Another possible extension would focus on the implications of different inventory management policies. Past research addresses this issue by studying the optimal inventory policies in the face of different types of advance demand information. But papers that simultaneously consider price and delivery lead time are just recently emerging.

While the model we have used is not overly complicated in itself, it captures the essential characteristics of pricing effects on market behavior and the results obtained unveil interesting managerial insights. One of the more important of these insights is the potential savings through reduced uncertainty and holding costs as a result of simply recognizing and acting on customer differences in flexibility. We also obtained some interesting insight on pricing decisions, and the importance of the proper demand model in such decisions. Under the additive model, the optimal MTS did not change under the LPDD option, while under the multiplicative model, where the optimal MTS price is a function of inventory-related cost parameters, the optimal MTS price decreases under the LPDD option if there is little differentiation in time sensitivity among customers. In general, however, the multiplicative demand model can lead to an increase or decrease in the MTS price. In both the additive demand cases, however, we saw that a high degree of time sensitivity in one subset of the market can actually provide additional surplus (through reduced discount prices) to customers who are willing to wait for demand satisfaction.

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Appendix A

A.1. Proofs for main properties

A.1.1. Proof of Property 2

\( \Pi_1 \) depends on \( y^i_0 \) through \( C^i \), which is convex in \( y^i_0 \). So \( \Pi_1 \) is concave in \( y^i_0 \). By the first-order optimality condition, the unique maximizer \( y^i_0^* \) satisfies \( F_{y^i_0} \left( \frac{\partial}{\partial y^i_0} \right) = \frac{k}{b_1+b_R} \). Part 2 follows from the monotonicity of \( F(\cdot) \). Part 3 is obtained by substituting Part 2 into \( C^i \) and using the definition of \( C_0(b_R, h) \). Part 4 just rearranges the expressions by our definitions of profit functions. \( \Box \)

A.1.2. Proof of Lemma 1

Under the more general market split coefficient functions, the first- and second-order derivatives of \( \Pi_1(\cdot) \) with respect to \( x_1 \) are

\[
\frac{\partial \Pi_1}{\partial x_1}(p^0_0, x_1, b_1, h) = R(p^0_0, b_1, h) \left[ (x_1 - k(p^0_0, b_1, h)) \frac{\partial x_1}{\partial x_1} + x_1(x_1) \right].
\]

\[
\frac{\partial^2 \Pi_1}{\partial x_1^2}(p^0_0, x_1, b_1, h) = R(p^0_0, b_1, h) \left[ 2 \frac{\partial x_1}{\partial x_1} + (x_1 - k(p^0_0, b_1, h)) \frac{\partial^2 x_1}{\partial x_1^2} \right].
\]

When \( 0 \leq x_1 < k(p^0_0, b_1, h) \), because \( \frac{\partial x_1}{\partial x_1} < 0 \) and \( x_1 > 0 \), we have \( \frac{\partial^2 \Pi_1}{\partial x_1^2} > 0 \). When \( x_1 = k(p^0_0, b_1, h) < 1 \),

\[
\frac{\partial \Pi_1}{\partial x_1} = R(p^0_0)x_1(k(p^0_0, b_1, h) > 0).
\]

When \( x_1 = 1 \), since \( x_1(1) = 0 \), we have

\[
\frac{\partial \Pi_1}{\partial x_1} = R(p^0_0)(1 - k(p^0_0, b_1, h)) \frac{\partial x_1}{\partial x_1} < 0.
\]

By concavity, there exists a unique \( x_1(p^0_0) \) in \( k(p^0_0, b_1, h), 1 \) that satisfies the first-order optimality condition. \( \Box \)

A.1.3. Proof of Property 4

By our definition of profit functions,

\[
\Pi_1(b_1, h) - \Pi_0(b, h) = [R - C_0(b_1, h)](1 - x_1^*) + Rx_1^*x_2^* - (R - C_0(b, h))
\leq [R - C_0(b_1, h)](1 - x_1^*) + Rx_1^*x_2^* - R + (1 - x_1^*)C_0(b_1, h) + x_1^*C_0(b_2, h)
= x_1^* \cdot R \cdot [x_1^* - k(p^0_1, b_2, h)].
\]

Since \( x_1^* > k(p^0_1, b_1, h) \) and \( x_1^* < 1 = k(p^0_1^*, b_1, h) \), by the monotonicity of \( k(p^0_1, b, h) \) in \( h \), there must exist \( 0 < b_2(b_1) < b_1 \) such that \( x_1^* - k(p^0_1^*, b_2(b_1), h) = 0 \). Then when \( 0 < b_2 < b_2(b_1), x_1^* < k(p^0_1^*, b_2(b_1), h) \) and the result follows. \( \Box \)

A.1.4. Proof of Property 6

That \( \Pi_1(p^0_0) \) is concave can be obtained from the non-positivity of its second-order derivative. By the first-order condition, \( p_0^0^* \) satisfies,

\[
\mu'(p^0_0^*)\left[ p_0^0^* - C_0(b_1, h) \right] + \mu(p^0_0^*) \mu + \frac{C_0^2(b_1, h)}{4p_0^0} \mu \left[ \mu(p^0_0^*) - \mu(p^0_0^*) \right] = 0
\]
Then we have
\[
\frac{\partial \Pi_i}{\partial p|_{p_i}^{\prime} = \mu'(p) \left[ C_0(b_2, h) - C_0(b_1, h) + \frac{C_0^2(b_1, h)}{4p\mu} \left( 2 - \frac{C_0(b_2, h)}{p\mu} \right) \right] = \mu'(p) g(b_1, b_2),
\]
where \( g(b_1, b_2) = C_0(b_2, h) - C_0(b_1, h) + \frac{C_0^2(b_1, h)}{4p\mu} \left( 2 - \frac{C_0(b_2, h)}{p\mu} \right) \). By our assumption that \( \Pi_i(b_2, h) \geq 0 \) for any effective \( b_2 \),
\[
\frac{C_0(b_2, h)}{p\mu} = \frac{\mu(p) C_0(b_2, h)}{p\mu(p)\mu} \leq 1.
\]
Then we have
\[
g(b_1, b_2) = C_0(b_2, h) - C_0(b_1, h) + \frac{C_0^2(b_1, h)}{4p\mu} \left( 2 - \frac{C_0(b_2, h)}{p\mu} \right) \\
\quad \geq C_0(b_2, h) - C_0(b_1, h) + \frac{C_0^2(b_1, h)}{4p\mu} \\
\quad > C_0(b_2, h) - C_0(b_1, h) + \frac{C_0^2(b_1, h) \mu(p_0(b_1, h))}{4p_0(b_1, h) \mu(p_0(b_1, h))} \\
\quad = C_0(b_2, h) - C_0(b_1, h) + \frac{1}{4} C_0(b_1, h) [ 1 - k(p_0(b_1, h)) ] = C_0(b_2, h) - C_0(b_1, h) \frac{3 + k(p_0(b_1, h))}{4} 
\]
Since \( C_0(0, h) = 0 \), \( g(b_1, 0) < 0 \). On the other hand, \( g(b_1, b_1) > 0 \). There must exist a \( b_2(b_1) < b_1 \) such that \( g(b_1, b_2(b_1)) = 0 \) and \( g(b_1, b_2) > 0 \) for any \( b_2 > b_2(b_1) \). That \( \mu'(p) < 0 \) implies \( \frac{\partial \Pi_i}{\partial p|_{p_i}^{\prime} > 0 \) for \( b_2 > b_2(b_1) \). The concavity of \( \Pi_i \) implies that \( p_0^i(b_1, h) < p_0^i(b_2, h) \) for \( b_2 \) sufficiently large. \( b \) in the base model is the weighted average of \( b_1 \) and \( b_2 \) and \( p_0^i \) is increasing in \( b \). We conclude that \( p_0^i(b_1, h) < p_0^i(b_2, h) \). For the other direction, we can follow the similar logic as above to show that
\[
g(b_1, b_2) < C_0(b_2, h) - C_0(b_1, h) \frac{1 + k(p_0(b_1, h))}{2}.
\]
Note that \( g(b_1, 0) < 0 \) and \( g(b_1, b_1) > 0 \). The monotonicity of \( C_0(b, h) \) in \( b \) implies that there exists another threshold value \( b_2' \) such that \( g(b_1, b_2') < 0 \) and \( p_0^i(b_1, h) > p_0^i(b_2, h) \) for \( 0 < b_2 < b_2' \). By the Maximization Theorem, \( p_0^i(b, h) \) is continuous in \( b \). By the Fixed-Point theorem there must exist a \( b_2' \) with \( b_2' < b_2' < b_2' \) such that \( p_0^i(b_1, h) = p_0^i(b_2', h), p_0^i(b_1, h) < p_0^i(b_2, h) \) for \( b_2 > b_2' \) and \( p_0^i(b_1, h) > p_0^i(b_2, h) \) for \( b_2 < b_2' \). Parts 2 and 3 thus follow. For Part 4, by the monotonicity of \( p_0^i(b, h) \) in \( b \), if \( p_0^i(b_1, h) > p_0^i(b_2, h) \), then there must exist \( 0 < b_2' < b_2' \) at which \( p_0^i(b_1, h) = p_0^i(b, h) \) and \( p_0^i(b_1, h) > p_0^i(b, h) \) for \( 0 < b_2 \leq b_2' \) all \( b_2 \). 

References


